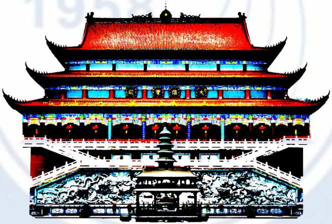
Relative phase between electric and magnetic Λ form factors

Rinaldo Baldini Ferroli, Alessio Mangoni and Simone Pacetti

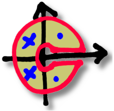


Workshop of the Baryon Production at BESIII

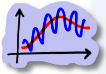
University of Science and Technology of China, Hefei, China

September 14th-16th, 2019

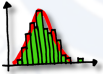
Agenda


Baryon form factors and dispersion relations

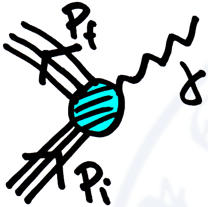

The dispersive approach for the the ratio G_E^Λ/G_M^Λ


Data and meaning of the phase determination


Technical details of the procedure


Results and final considerations

Baryon-photon vertex



Baryon electromagnetic four-current ($q = p_f - p_i$)

$$\langle P_f | J^\mu_{\mathrm{EM}}(0) | P_i \rangle = e\,\overline{u}(p_f)\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2M_p} F_2(q^2)\right] u(p_i)$$

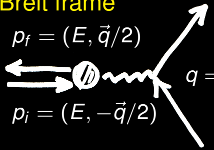
$F_1(q^2)$ and $F_2(q^2)$ are the Dirac and Pauli form factors

$$F_1(0) = \mathcal{Q}_{\mathcal{B}} \qquad F_2(0) = \kappa_{\mathcal{B}}$$

$\mathcal{Q}_{\mathcal{B}}$ = electric charge $\qquad \kappa_{\mathcal{B}}$ = anomalous magnetic moment

Breit frame

$p_f = (E, \vec{q}/2)$



$q = (0, \vec{q})$

$p_i = (E, -\vec{q}/2)$

$$\langle P_f | J^\mu_{\mathrm{EM}}(0) | P_i \rangle \equiv J^\mu_{\mathrm{EM}} = \left(J^0_{\mathrm{EM}}, \vec{J}_{\mathrm{EM}}\right)$$

- $J^0_{\mathrm{EM}} = e\left(F_1(q^2) + \dfrac{q^2}{4M_p^2} F_2(q^2)\right)$
- $\vec{J}_{\mathrm{EM}} = e\,\overline{u}(p_f)\vec{\gamma} u(p_i)\left(F_1(q^2) + F_2(q^2)\right)$

Sachs form factors

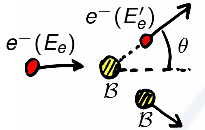
- $G_E(q^2) = F_1(q^2) + \dfrac{q^2}{4M_{\mathcal{B}}^2} F_2(q^2)$
- $G_M(q^2) = F_1(q^2) + F_2(q^2)$

Normalizations

- $G_E(0) = \mathcal{Q}_{\mathcal{B}}$
- $G_M(0) = \mu_{\mathcal{B}} = \kappa_{\mathcal{B}} + \mathcal{Q}_{\mathcal{B}}$

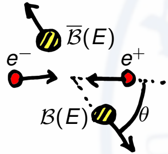
$\mu_{\mathcal{B}}$ = total magnetic moment

Cross sections and Coulomb correction



Elastic scattering cross section (Rosenbluth)

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 E'_e \cos^2\left(\frac{\theta}{2}\right)}{4E_e^3 \sin^4\left(\frac{\theta}{2}\right)} \left[G_E^2 - \tau\left(1+2(1-\tau)\tan^2\left(\frac{\theta}{2}\right)\right) G_M^2\right] \frac{1}{1-\tau}$$



Annihilation cross section

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta\, \mathcal{C}}{16E^2}\left[\left(1+\cos^2(\theta)\right)|G_M|^2 + \frac{1}{\tau}\sin^2(\theta)\,|G_E|^2\right]$$

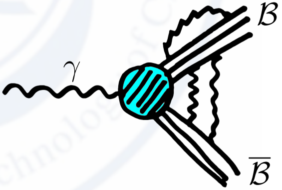
$$\tau = E^2/M_{\mathcal{B}}^2$$

$$\beta = \sqrt{1-1/\tau}$$

Coulomb correction

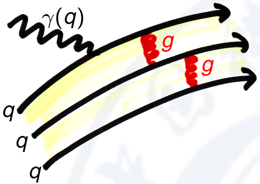
$$\mathcal{C} = \frac{\pi\alpha/\beta}{1-e^{-\pi\alpha/\beta}}$$

- $p\bar{p}$ Coulomb interaction as FSI
- Only S-wave



pQCD asymptotic behavior
Space-like region

V.A. Matveev, R.M. Muradian, A.N. Tavkhelidze, LNC7 (1973) 719
S. J. Brodsky, G. R. Farrar, PRL31 (1973) 1153
M. V. Galynsky, E. A. Kuraev JETPL96 (2012) 6



- **pQCD**: as $q^2 \to -\infty$, F_1, F_2, G_E, G_M follow power laws driven by counting rules.
- Valence quarks exchange gluons to distribute the photon momentum transfer q.

Non-helicity-flip current $J^{\lambda,\lambda}(q^2)$

- $J^{\lambda,\lambda}(q^2) \propto G_M(q^2)$
- Two gluon propagators
- $G_M(q^2) \underset{q^2\to-\infty}{\sim} \left(q^2\right)^{-2}$

Helicity-flip current $J^{\lambda,-\lambda}(q^2)$

- $J^{\lambda,-\lambda}(q^2) \propto G_E(q^2)/\sqrt{-q^2}$
- [Two gluon propagators]$/\sqrt{-q^2}$
- $G_E(q^2) \underset{q^2\to-\infty}{\sim} \left(q^2\right)^{-2}$

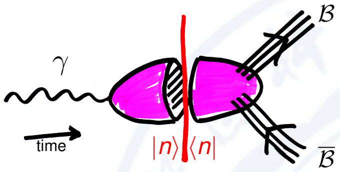
Dirac and Pauli form factors

- $F_1(q^2) \underset{q^2\to-\infty}{\sim} \left(q^2\right)^{-2}$
- $F_2(q^2) \underset{q^2\to-\infty}{\sim} \left(q^2\right)^{-3}$

Ratio of Sachs form factors

- $\dfrac{G_E(q^2)}{G_M(q^2)} \underset{q^2\to-\infty}{\sim}$ constant

Baryon form factors
Time-like region ($q^2 > 0$)



- Crossing symmetry:

$$\langle P(p')|j^\mu|P(p)\rangle \to \langle \overline{P}(p')P(p)|j^\mu|0\rangle$$

- Form factors are complex functions of q^2

Optical theorem

$$\mathrm{Im}\langle \overline{P}(p')P(p)|j^\mu|0\rangle \sim \sum_n \langle \overline{P}(p')P(p)|j^\mu|n\rangle\langle n|j^\mu|0\rangle \implies \begin{cases} \mathrm{Im}F_{1,2} \neq 0 \\ \text{for } q^2 > 4M_\pi^2 \end{cases}$$

$|n\rangle$ are on-shell intermediate states: 2π, 3π, 4π, ...

Time-like asymptotic behavior

Phragmèn Lindelöf theorem

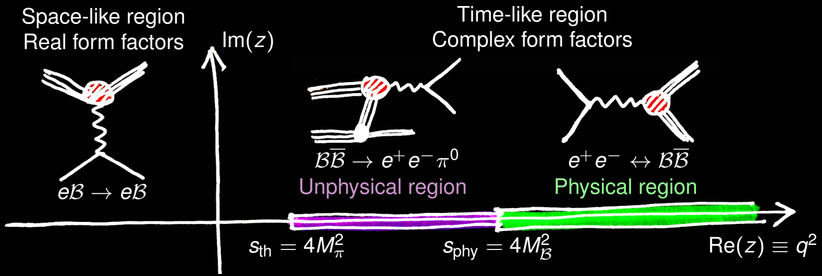
If $\boldsymbol{f(z)} \longrightarrow \boldsymbol{a}$ as $z \to \infty$ along a straight line, and $\boldsymbol{f(z)} \longrightarrow \boldsymbol{b}$ as $z \to \infty$ along another straight line, and $f(z)$ is regular and bounded in the angle between, then $\boldsymbol{a} = \boldsymbol{b}$ and $\boldsymbol{f(z)} \longrightarrow \boldsymbol{a}$ **uniformly in this angle**.

- $$\underbrace{\lim_{q^2 \to -\infty} G_{E,M}(q^2)}_{\text{space-like}} = \underbrace{\lim_{q^2 \to +\infty} G_{E,M}(q^2)}_{\text{time-like}}$$

- $$G_{E,M} \underset{q^2 \to +\infty}{\sim} (q^2)^{-2}$$ **Must be real**

Analyticity of form factors

q^2-complex plane

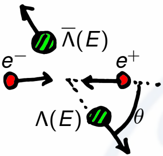


Only the real axis of the q^2-complex plane is experimentally accessible

Space-like region $q^2 < 0$	Time-like region* $s_{\text{th}} < q^2 \leq s_{\text{phy}}$	Time-like region $q^2 \geq s_{\text{phy}}$	
$e\mathcal{B} \to e\mathcal{B}$	$\mathcal{B}\overline{\mathcal{B}} \to e^+e^-\pi^0$	$e^+e^- \leftrightarrow \mathcal{B}\overline{\mathcal{B}}$	$e^+e^- \leftrightarrow \mathcal{B}\overline{\mathcal{B}}$ (pol.)
G_E, G_M	$\lvert G_E\rvert$, $\lvert G_M\rvert$	$\lvert G_E\rvert$, $\lvert G_M\rvert$	$\lvert G_E\rvert$, $\lvert G_M\rvert$, $\arg(G_E/G_M)$

***In case of $\mathcal{B} = p$**: C. Adamuscin, E.A. Kuraev, E. Tomasi-Gustafsson, F. Maas PRC75, 045205
E. A. Kuraev et al., JETP115, 93
G. I. Gakh, E. Tomasi-Gustafsson, A. Dbeyssi, A.G. Gakh PRC86, 025204

Λ form factors



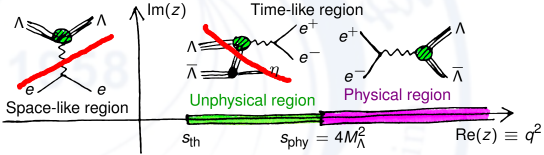
Annihilation cross section

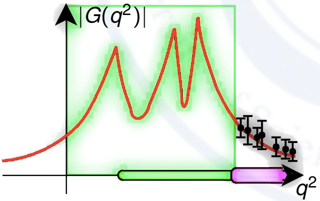
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta \not{C}}{16E^2}\left[\left(1+\cos^2(\theta)\right)|G_M^\Lambda|^2 + \frac{1}{\tau}\sin^2(\theta)\,|G_E^\Lambda|^2\right]$$

$$\tau = E^2/M_\Lambda^2$$

$$\beta = \sqrt{1 - 1/\tau}$$

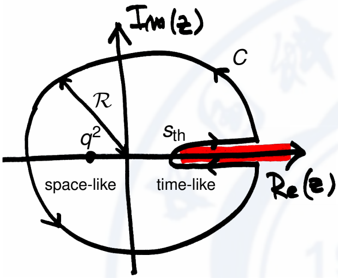
- Theoretical threshold at $s_{\mathrm{th}} = (2M_\pi + M_{\pi^0})^2$.
- Difficult to measure in space-like and unphysical regions.
- Relative phase from weak decay.





- Unitarity: only isoscalar intermediate states contributions.
- Form factors have not vanishing imaginary part above the theoretical threshold.
- The electric form factor vanishes at $q^2 = 0$.

Dispersion relations



- The form factors are analytic on the q^2-plane with a multiple cut (s_{th}, ∞).
- Dispersion relation for the imaginary part $(q^2 < 0)$

$$G(q^2) = \lim_{\mathcal{R}\to\infty} \frac{1}{2\pi i}\oint_C \frac{G(z)dz}{z-q^2} = \frac{1}{\pi}\int_{s_{th}}^{\infty}\frac{\mathrm{Im}[G(s)]ds}{s-q^2}$$

- Dispersion relation for the logarithm $(q^2 < 0)$
  B.V. Geshkenbein, Yad. Fiz. 9 (1969) 1232.

$$\ln[G(q^2)] = \frac{\sqrt{s_{th}-q^2}}{\pi}\int_{s_{th}}^{\infty}\frac{\ln[|G(s)|]ds}{(s-q^2)\sqrt{s-s_{th}}}$$

Experimental inputs

- Space-like data on the real values of form factors from: $e\mathcal{B} \to e\mathcal{B}$ and $e^{\uparrow}\mathcal{B} \to e^{-}\mathcal{B}^{\uparrow}$, with polarization.
- Time-like data on form factor moduli from: $e^+e^- \leftrightarrow \mathcal{B}\overline{\mathcal{B}}$.
- Time-like data on G_E/G_M phase from: $e^+e^- \leftrightarrow \mathcal{B}^{\uparrow}\overline{\mathcal{B}}$ (polarization).

Theoretical ingredients

- Analyticity $\Rightarrow$ convergence relations.
- Normalization and threshold values.
- Asymptotic behavior $\Rightarrow$ super-convergence relations

Advantages and drawbacks of dispersive approaches

Advantages

- DR's are based on unitarity and analyticity $\Rightarrow$ model-independent approach.
- DR's relate data from different processes in different energy regions

$$\left[\begin{array}{c}\textbf{space-like}\\ \textbf{form factor}\\ e\mathcal{B}\to e\mathcal{B}\end{array}\right] = \int_{s_{\rm th}}^{\infty}\left[\begin{array}{c}\textbf{Im(form factor) or ln|form factor|}\\ \textbf{over the time-like cut } (s_{\rm th},\infty)\\ e^+e^-\to\mathcal{B}\overline{\mathcal{B}} + \textbf{theory}\end{array}\right]$$

- Normalizations and theoretical constraints can be directly implemented.
- Form factors can be computed in the whole q^2-complex plane.

Drawbacks

- Very long-range integration

Remedy #1
pQCD power laws

Remedy #2
Subtracted DR's

- No data in the unphysical region, crucial in dispersive analyses.

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- Normalizations and theoretical constraints can be directly implemented.
- Form factors can be computed in the whole q^2-complex plane.
- Poles cancel out in the ratio!

Drawbacks

- Very long-range integration

Remedy #1
pQCD power laws

Remedy #2
Subtracted DR's

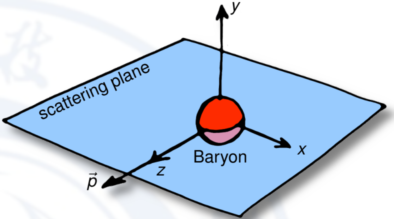
- No data in the unphysical region, crucial in dispersive analyses.

Polarization in the time-like region

The ratio $R(q^2)$ is complex for $q^2 \geq s_{\rm th}$

$$\frac{G_E(q^2)}{G_M(q^2)} = \frac{|G_E(q^2)|}{|G_M(q^2)|} e^{i\rho(q^2)}$$

The polarization depends on the phase ρ



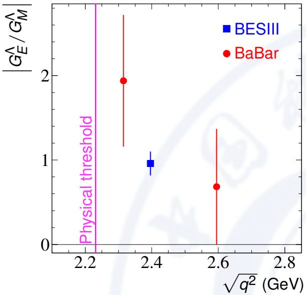
[A.Z. Dubnickova, S. Dubnicka, M.P. Rekalo, NCA109,241(96)]

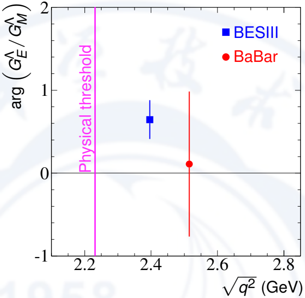
- $$\left.\mathcal{P}_y = -\frac{\sin(2\theta)\sin(\rho)}{D\sqrt{\tau}}\frac{|G_E|}{|G_M|} = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} \equiv \mathcal{A}_y\right\}$$ Does not depend on P_e
- $$\mathcal{P}_x = -P_e \frac{2\sin(2\theta)\cos(\rho)}{D\sqrt{\tau}}\frac{|G_E|}{|G_M|}$$
- $$\left.\mathcal{P}_z = P_e \frac{2\cos(\theta)}{D}\right\}$$ Does not depend on ρ

$$D = 1 + \cos^2(\theta) + \frac{|G_E|^2}{|G_M|^2}\frac{\sin^2(\theta)}{\tau} \qquad \tau = \frac{q^2}{4M_{\mathcal{B}}^2}$$

- P_e is the electron polarization
- θ is the scattering angle

Data on modulus and phase of $G_E^\Lambda / G_M^\Lambda$



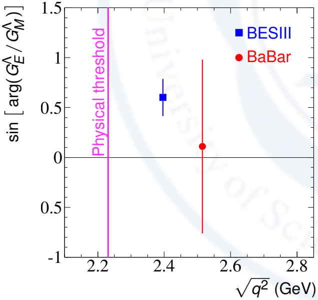


BaBar 2007

Phys. Rev. D 76, 092006

BESIII 2019

J. Phys.: Conf. Ser. 1137, 012010

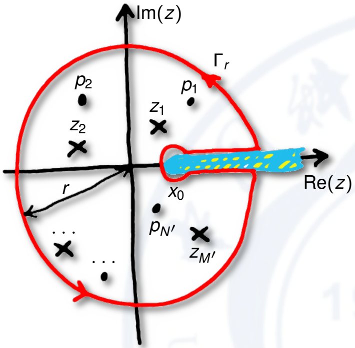


$$\mathcal{P}_y = -\frac{2M_\Lambda \sqrt{q^2}\,\sin(2\theta)\,|G_E^\Lambda / G_M^\Lambda|\,\sin\left[\arg(G_E^\Lambda / G_M^\Lambda)\right]}{q^2\left(1+\cos^2(\theta)\right) + 4M_\Lambda^2 |G_E^\Lambda / G_M^\Lambda|^2 \sin^2(\theta)}$$

- Polarization $\rightarrow$ sinus of the relative phase.
- Spin correlation $\rightarrow$ cosinus of the relative phase.
- There are no information about the determination of the relative phase.

Is the determination meaningful?

On the meaning of the phase determination



- Consider a function $R(z)$ with N poles $\{p_j\}_{j=1}^{N}$, M zeros $\{z_k\}_{k=1}^{M}$ and the positive real cut (x_0, ∞).
- Residue theorem:

$$\lim_{r\to\infty}\frac{1}{2i\pi}\oint_{\Gamma_r}\frac{d\ln\left[R(z)\right]}{dz}dz = M - N.$$

- By considering single contributions:

$$\lim_{r\to\infty}\frac{1}{2i\pi}\oint_{\Gamma_r}\frac{d\ln\left[R(z)\right]}{dz}dz = \frac{\arg\left[R(\infty)\right]-\arg\left[R(x_0)\right]}{\pi}.$$

$$\boxed{\arg\left[R(\infty)\right]-\arg\left[R(x_0)\right] = \pi(M-N)}$$

- From factors are analytic in the q^2 complex plane with the positive real cut $(s_{\rm th}, \infty)$.
- **By assuming no zero for G_M^Λ**, the ratio G_E^Λ/G_M^Λ is analytic in the same domain.
- Form factors and hence their ratio G_E^Λ/G_M^Λ are real for real values of q^2 with $q^2 \notin (s_{\rm th}, \infty)$.

$$\lim_{q^2\to s_{\rm th}^-}\arg\left[\frac{G_E^\Lambda(q^2)}{G_M^\Lambda(q^2)}\right] = \begin{cases} 0 & G_E^\Lambda/G_M^\Lambda > 0 \text{ as } q^2\to s_{\rm th}^- \\ \pm\pi & G_E^\Lambda/G_M^\Lambda < 0 \text{ as } q^2\to s_{\rm th}^- \end{cases}$$

Dispersive procedure

The ratio: $R(q^2) \equiv \dfrac{G_E^\Lambda(q^2)}{G_M^\Lambda(q^2)}$, since $G_E^\Lambda(0) = 0$ $\Rightarrow$ $\begin{cases} R(0) = 0 \\ R(s_{\rm phy}) = 1 \end{cases}$

Electric and magnetic form factors have the same asymptotic behavior

$$\Rightarrow \quad R(q^2) = \frac{G_E^\Lambda(q^2)}{G_M^\Lambda(q^2)} = O(1) \text{ as } q^2 \to \pm\infty.$$

Dispersion relations for the **imaginary** and **real** part subtracted at $q^2 = 0$:

$$R(q^2) = R(0) + \frac{q^2}{\pi}\int_{s_{\rm th}}^{\infty} \frac{\mathrm{Im}\,[R(s)]}{s(s-q^2)} ds = \frac{q^2}{\pi}\int_{s_{\rm th}}^{\infty} \frac{\mathrm{Im}\,[R(s)]}{s(s-q^2)} ds, \quad \forall\, q^2 \notin (s_{\rm th}, \infty);$$

$$\mathrm{Re}[R(q^2)] = \frac{q^2}{\pi}\,\mathrm{Pr}\int_{s_{\rm th}}^{\infty} \frac{\mathrm{Im}\,[R(s)]}{s(s-q^2)} ds, \qquad \forall\, q^2 \in (s_{\rm th}, \infty)^+.$$

The subtraction automatically provides the normalization at $q^2 = 0$.

The parametrization for $R(s)$

The imaginary part of the ratio is parametrized as a combination of Chebyshev polynomials $T_j(x)$.

$$\mathrm{Im}\left[R(s)\right] \equiv Y(s;\vec{\alpha}) = \begin{cases} \displaystyle\sum_{j=0}^{P-1} \alpha_j T_j\left[x(s)\right] & s_{\mathrm{th}} < s < s_{\mathrm{asy}} \\ 0 & s \geq s_{\mathrm{asy}} \end{cases}$$

$$x(s) = 2\frac{s - s_{\mathrm{th}}}{s_{\mathrm{asy}} - s_{\mathrm{th}}} - 1$$

$$x(s) \in [-1, 1]$$

Theoretical conditions on $Y(s;\vec{\alpha})$

- $R(s_{\mathrm{th}})$ is real $\Rightarrow Y(s_{\mathrm{th}};\vec{\alpha}) = 0$
- $R(s_{\mathrm{phy}})$ is real $\Rightarrow Y(s_{\mathrm{phy}};\vec{\alpha}) = 0$
- $R(s \geq s_{\mathrm{asy}})$ is real $\Rightarrow Y(s \geq s_{\mathrm{asy}};\vec{\alpha}) = 0$

Theoretical conditions on $\mathrm{Re}\left[R(s)\right]$

- $\mathrm{Re}\left[R(s_{\mathrm{phy}})\right] = \dfrac{s_{\mathrm{phy}}}{\pi}\mathrm{Pr}\displaystyle\int_{s_{\mathrm{th}}}^{s_{\mathrm{asy}}} \frac{Y(s;\vec{\alpha})}{s(s-s_{\mathrm{phy}})}ds = 1$
- $\mathrm{Re}\left[R(s_{\mathrm{asy}})\right] = \dfrac{s_{\mathrm{asy}}}{\pi}\mathrm{Pr}\displaystyle\int_{s_{\mathrm{th}}}^{s_{\mathrm{asy}}} \frac{Y(s;\vec{\alpha})}{s(s-s_{\mathrm{asy}})}ds = \pm 1$

Experimental conditions in the time-like region ($s > s_{\mathrm{phy}}$)

- $\mathbf{sin}\left[\mathbf{arg}\left(\boldsymbol{R(s)}\right)\right]$: One data point from BESIII and one data point from BaBar.
- $|\boldsymbol{R(s)}|$: One data point from BESIII and two data points from BaBar.

The χ^2 definition

$$\chi^2(\vec{\alpha}) = \chi^2_R(\vec{\alpha}) + \chi^2_\phi(\vec{\alpha}) + \tau_{\rm phy}\chi^2_{\rm phy}(\vec{\alpha}) + \tau_{\rm asy}\chi^2_{\rm asy}(\vec{\alpha}) + \tau_{\rm curv}\chi^2_{\rm curv}(\vec{\alpha})$$

Data set $\{s_j, |R_j|, \delta|R_j|\}_{j=1}^{M} \longrightarrow \chi^2_R(\vec{\alpha}) = \sum_{j=1}^{M}\left(\frac{\sqrt{X(s_j;\vec{\alpha})^2 + Y(s_j;\vec{\alpha})^2} - |R_j|}{\delta|R_j|}\right)^2$ $\quad X(s;\vec{\alpha}) = \mathrm{Re}[R(s)]$

Data set $\{s_k, \sin(\phi_k), \delta\sin(\phi_k)\}_{k=1}^{P} \longrightarrow \chi^2_\phi(\vec{\alpha}) = \sum_{k=1}^{P}\left(\frac{\sin\left[\arctan\left(\frac{Y(s_k;\vec{\alpha})}{X(s_k;\vec{\alpha})}\right)\right] - \sin(\phi_k)}{\delta\sin(\phi_k)}\right)^2$

Constraint at $q^2 = s_{\rm phy} \longrightarrow \chi^2_{\rm phy}(\vec{\alpha}) = \left(1 - X(s_{\rm phy};\vec{\alpha})\right)^2$

Constraint at $q^2 = s_{\rm asy} \longrightarrow \chi^2_{\rm asy}(\vec{\alpha}) = \left(1 - X(s_{\rm asy};\vec{\alpha})^2\right)^2$

The parameters $\tau_{\rm phy}$ and $\tau_{\rm asy}$ are chosen in order to nullify the corresponding χ^2 so that the condition is exactly fulfilled.

Oscillation damping $\longrightarrow \chi^2_{\rm curv}(\vec{\alpha}) = \int_{s_{\rm th}}^{s_{\rm asy}}\left(\frac{d^2 Y(s;\vec{\alpha})}{ds^2}\right)^2 ds$

The integral equation obtained by the dispersion relations is an ill-posed problem whose solution has to be regularized.

The regularization parameter $\tau_{\rm curv}$ is chosen in order to attenuate spurious oscillations.

- Too large values would cancel physical information.
- Too small values would leave a high level of noise.

Our parametrization

$$\mathrm{Im}\left[R(s)\right] \equiv Y(s;\vec{\alpha},s_{\mathrm{asy}}) = \begin{cases} \displaystyle\sum_{j=0}^{P-1} \alpha_j T_j\left[x(s)\right] & s_{\mathrm{th}} < s < s_{\mathrm{asy}} \\ 0 & s \geq s_{\mathrm{asy}} \end{cases}$$

$$x(s) = 2\,\frac{s - s_{\mathrm{th}}}{s_{\mathrm{asy}} - s_{\mathrm{th}}} - 1$$

$$x(s) \in [-1, 1]$$

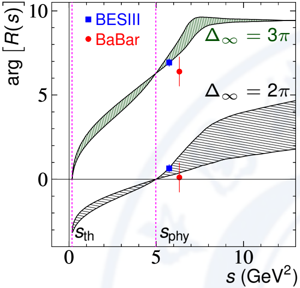
- Theoretical conditions: $Y(s_{\mathrm{th}};\vec{\alpha},s_{\mathrm{asy}}) = Y(s_{\mathrm{phy}};\vec{\alpha},s_{\mathrm{asy}}) = Y(s_{\mathrm{asy}};\vec{\alpha},s_{\mathrm{asy}}) = 0$ set the three parameters: $\boldsymbol{\alpha_1}$, $\boldsymbol{\alpha_2}$, $\boldsymbol{\alpha_3}$.
- The asymptotic threshold s_{asy} is left as a free parameter.
- By considering P Chebyshev polynomials there are **(P − 3)** free parameters.
- We have used $\boldsymbol{P = 6}$ and hence **three** free parameters: $\boldsymbol{\alpha_4}$, $\boldsymbol{\alpha_5}$ and $\boldsymbol{s_{\mathrm{asy}}}$.

$\boldsymbol{\tau_{\mathrm{phy}} = 10^2}$ ← The real part of $R(s)$ is constrained to one at $s = s_{\mathrm{phy}}$.

$\boldsymbol{\tau_{\mathrm{asy}} = 0}$ ← **No constraint** for the real part of $R(s)$ at $s = s_{\mathrm{asy}}$.

$\boldsymbol{\tau_{\mathrm{curv}} = 5 \cdot 10^{-4}}$ ← Low-degree polynomials do not need strong regularization.

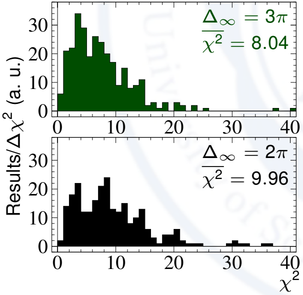
Phases and χ^2's



By collecting information on modulus and phase the dispersive procedure allows to establish the **determination of the phase**.

The lack of experimental information gives **two** possible solutions for the phase.

Different classes of solutions are characterized by the values of the phase at the theoretical and physical thresholds, $s_{\rm th}$, $s_{\rm phy}$, as well as at **infinity**.



The error bands are determined through a statistical analysis of the solutions obtained by repeating the minimization procedure on different sets of data generated by Gaussian fluctuations of the original ones.

The solutions are classified by the quantity

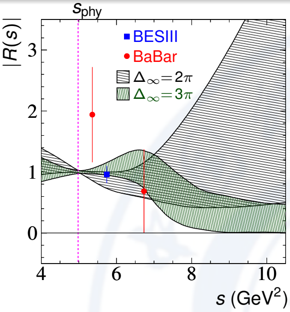
$$\Delta_\infty \equiv \arg\left[R(\infty)\right] - \arg\left[R(s_{\rm th})\right].$$

Only solutions with $\Delta_\infty = 3\pi$ and $\Delta_\infty = 2\pi$ are obtained with probabilities

$P(3\pi) = (54 \pm 3)\%$

$P(2\pi) = (46 \pm 3)\%$

Time-like moduli and space-like real parts

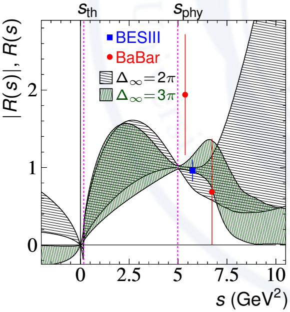


$\Delta_\infty = 2\pi$

- The first BaBar point is not fitted.
- At high s the errors diverge.
- $|R(s)|$ seems to increase with s.

$\Delta_\infty = 3\pi$

- The first BaBar point is not fitted.
- At high s the errors are stable.
- $|R(s)|$ seems to decrease with s.



$\Delta_\infty = 2\pi$

The ratio $R(s)$ has two space-like zeros at $s = 0$, as expected, and at $s \lesssim s_{\rm th}$.

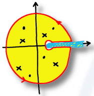
$\Delta_\infty = 3\pi$

The ratio $R(s)$ has "apparently" the only expected space-like zero at $s = 0$.

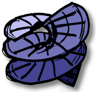
The value $\Delta_\infty = 3\pi$ does imply that such a zero has order three.

Final considerations

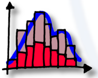
BESIII measured with unprecedented accuracy the **modulus** and the **phase** of the ratio G_E^Λ/G_M^Λ of electric and magnetic Λ form factors.



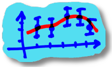
The BESIII measurement and also older, less precise data, can be analyzed by means of a **dispersive procedure** based on analyticity and a set of **first-principle constraints**.



The ability of the dispersive procedure to determine the **complex structure** of the ratio is limited by the lack of data.
BESIII measured modulus and phase at only one energy point.



Two classes of solutions are obtained. In both cases, time-like and space-like behaviors show interesting properties:
space-like zeros or unexpected **large determinations** for the phase.



More data at **different energies** would be crucial to enhance the predictive power of the dispersive procedure.

Thank you