Helicity analysis of semileptonic Λ decays

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Introduction

- Helicity method allows:
  - Compact calculations of the angular decay distributions
  - Analyze the semileptonic decays of polarized hyperon
  - Take into account the lepton mass effects

    $\Rightarrow$ vector and axial-vector currents
- **Aim**: a general modular method for the semileptonic hyperon decays
- Analysis is based on:
  - Helicity analysis for $\Xi^0 \to \Sigma^+(\to p\pi^0)l^-\bar{\nu}_l$ $(l = e^-, \mu^-)$ [EPJ C59 (2009) 27]
  - Polarization observables in e^+e^- annihilation to a $B\bar{B}$ pair [PRD 99 (2019) 056008]

Production process of two spin-$\frac{1}{2}$ baryons

- General framework of the $e^+e^- \to J/\psi \to \Lambda\bar{\Lambda}$ is described in [PRD99 (2019) 056008]
- Production process doesn't depend on the final states. It is the same for:
  - $e^+e^- \to J/\psi \to \Lambda\bar{\Lambda}$ with $\Lambda \to p\pi^-$ and $\bar{\Lambda} \to \bar{p}\pi^+$
  - $e^+e^- \to J/\psi \to \Lambda\bar{\Lambda}$ with $\Lambda \to pe^-\bar{\nu}_e$ and $\bar{\Lambda} \to \bar{p}\pi^+$

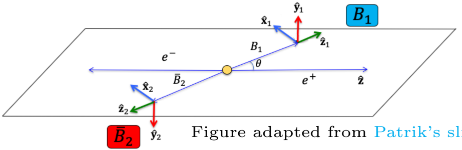


Figure adapted from Patrik's slides

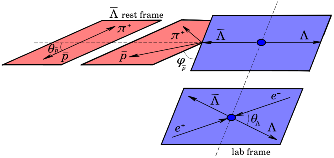
- Spin density matrix of the production process:

$$\rho_{B_1,\bar{B}_2} = \frac{1}{4}\sum_{\mu,\bar{\nu}=0}^{3} C_{\mu\bar{\nu}}(\theta_1)\sigma_\mu^{B_1} \otimes \sigma_{\bar{\nu}}^{\bar{B}_2}$$

$$\begin{aligned}
C_{00} &= 2(1+\alpha_\psi \cos^2\theta_1),\\
C_{02} &= 2\sqrt{1-\alpha_\psi^2}\sin\theta_1\cos\theta_1\sin(\Delta\Phi),\\
C_{11} &= 2\sin^2\theta_1,\\
C_{13} &= 2\sqrt{1-\alpha_\psi^2}\sin\theta_1\cos\theta_1\cos(\Delta\Phi),\\
C_{20} &= -C_{02},\\
C_{22} &= \alpha_\psi C_{11},\\
C_{31} &= -C_{13},\\
C_{33} &= -2(\alpha_\psi+\cos^2\theta_1).
\end{aligned}$$

- Main parameters: α_ψ, $\Delta\Phi$

Decay chain: $\frac{1}{2} \to \frac{1}{2} + 0$ (1)



- Full determination in [PRD99 (2019) 056008]
- Decay matrix or transition matrix $a_{\mu\nu}$

$$\sigma_\mu \to \sum_{\nu=0}^{3} a_{\mu\nu} \sigma_\nu^d$$

- $J = 1/2$ hyperon ($\bar{\Lambda}$) decays into a $J = 1/2$ baryon ($\bar{p}$) and a $J = 0$ pseudoscalar (π^+)

$$a_{\mu\nu} = \frac{1}{4\pi} \sum_{\lambda,\lambda'=-1/2}^{1/2} B_\lambda B^*_{\lambda'} \sum_{\kappa,\kappa'=-1/2}^{1/2} (\sigma_\mu)^{\kappa,\kappa'} (\sigma_\nu)^{\lambda',\lambda} \mathcal{D}^{1/2*}_{\kappa,\lambda}(\Omega) \mathcal{D}^{1/2}_{\kappa',\lambda'}(\Omega)$$

- B_λ, $B_{\lambda'}$ - helicity amplitudes, $\{B_{\frac{1}{2}}, B_{-\frac{1}{2}}\}$
- κ, κ' - index of mother hyperon ($\bar{\Lambda}$); λ, λ' - index of daughter baryon ($\bar{p}$)
- $\Omega = \{\phi, \theta, 0\}$

Decay chain: $\frac{1}{2} \to \frac{1}{2} + 0$ (2)

- Relation between helicity amplitudes and decay parameters

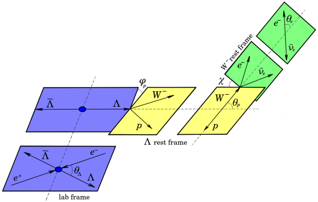
$$\text{normalization } |A_S|^2 + |A_P|^2 = |B_{-\frac{1}{2}}|^2 + |B_{\frac{1}{2}}|^2 = 1,$$
$$\alpha_D = -2\Re(A_S^* A_P) = |B_{1/2}|^2 - |B_{-1/2}|^2,$$
$$\beta_D = -2\Im(A_S^* A_P) = 2\Im(B_{1/2} B_{-1/2}^*),$$
$$\gamma_D = |A_S^*|^2 - |A_P|^2 = 2\Re(B_{1/2} B_{-1/2}^*),$$
$$\text{where } \beta_D = \sqrt{1-\alpha_D^2}\sin\varphi_D \text{ and } \gamma_D = \sqrt{1-\alpha_D^2}\cos\varphi_D$$

- Non-zero elements of the decay matrix $a_{\mu\nu}$

$$\begin{aligned}
a_{00} &= 1, & a_{21} &= \beta_D \cos\varphi + \gamma_D \cos\theta \sin\varphi, \\
a_{03} &= \alpha_D, & a_{22} &= \gamma_D \cos\varphi - \beta_D \cos\theta \sin\varphi, \\
a_{10} &= \alpha_D \cos\varphi \sin\theta, & a_{23} &= \sin\theta \sin\varphi, \\
a_{11} &= \gamma_D \cos\theta \cos\varphi - \beta_D \sin\varphi, & a_{30} &= \alpha_D \cos\theta, \\
a_{12} &= -\beta_D \cos\theta \cos\varphi - \gamma_D \sin\varphi, & a_{31} &= -\gamma_D \sin\theta, \\
a_{13} &= \sin\theta \cos\varphi, & a_{32} &= \beta_D \sin\theta, \\
a_{20} &= \alpha_D \sin\theta \sin\varphi, & a_{33} &= \cos\theta
\end{aligned}$$

- Main parameters: α_D, ϕ_D

Decay chain: $\frac{1}{2} \rightarrow \frac{1}{2} + \{0, \pm 1\}$ (1)



- Decay matrix or transition matrix $b_{\mu\nu}$

$$\sigma_\mu \rightarrow \sum_{\nu=0}^{3} b_{\mu\nu} \sigma_\nu^{d-W}$$

- $J = 1/2$ hyperon (Λ) decays into a $J = 1/2$ baryon (p) and a $J = \{0, \pm 1\}$ W^--boson ($\rightarrow l\nu_l$)

$$b_{\mu\nu} = \frac{1}{4\pi} \sum_{\lambda_2=-1/2}^{1/2} \sum_{\lambda_W,\lambda'_W=-1}^{1} H_{\lambda_2\lambda_W} H^*_{\lambda_2\lambda'_W} \sum_{\kappa,\kappa'=-1/2}^{1/2} (\sigma_\mu)^{\kappa,\kappa'} (\sigma_\nu)^{\lambda_2-\lambda'_W,\lambda_2-\lambda_W} \mathcal{D}^{1/2*}_{\kappa,\lambda_2-\lambda_W}(\Omega) \mathcal{D}^{1/2}_{\kappa',\lambda_2-\lambda'_W}(\Omega) \times$$

$$\sum_{\lambda_l,\lambda_\nu=-1/2}^{1/2} |h^l_{\lambda_l\lambda_\nu=\pm 1/2}|^2 \mathcal{D}^{1*}_{\lambda_W,\lambda_l-\lambda_\nu}(\Omega') \mathcal{D}^{1}_{\lambda'_W,\lambda_l-\lambda_\nu}(\Omega'),$$

- H_{λ_2,λ_W}, H_{λ_2,λ'_W} - helicity amplitudes, $\{H_{\frac{1}{2}0}, H_{-\frac{1}{2}0}, H_{\frac{1}{2}1}, H_{-\frac{1}{2}-1}\}$
- κ, κ' - index of mother hyperon (Λ); λ_2 - index of daughter baryon (p)
- λ_W, λ'_W - index of W^--boson; λ_l, λ_ν - index of lepton and neutrino
- $\Omega = \{\varphi, \theta, 0\}$, $\Omega' = \{\chi, \theta_l, 0\}$

Helicity amplitudes of the lepton pair $h^l_{\lambda_l \lambda_\nu}$

- Lepton and antineutrino spinors

$$\bar{u}_{l^-}(\mp\frac{1}{2}, p_{l^-}) = \sqrt{E_l + m_l}\left(\chi^\dagger_\mp, \frac{\pm|\vec{p}_{l^-}|}{E_l + m_l}\chi^\dagger_\mp\right),$$
$$v_{\bar{\nu}}(\frac{1}{2}) = \sqrt{E_\nu}\begin{pmatrix}\chi_+ \\ -\chi_+\end{pmatrix},$$

where $\chi_+ = \begin{pmatrix}1\\0\end{pmatrix}$ and $\chi_- = \begin{pmatrix}0\\1\end{pmatrix}$ are Pauli two-spinors

- SM form of the lepton current ($\lambda_W = \lambda_{l^-} - \lambda_{\bar{\nu}}$)

$$h^l_{\lambda_{l^-}=\mp 1/2, \lambda_{\bar{\nu}}=1/2} = \bar{u}_{l^-}(\mp\frac{1}{2})\gamma^\mu(1+\gamma_5)v_{\bar{\nu}}(\frac{1}{2})\begin{Bmatrix}\epsilon_\mu(-1)\\ \epsilon_\mu(0)\end{Bmatrix}$$

where $\epsilon^\mu(0) = (0;0,0,1)$ and $\epsilon^\mu(\mp 1) = (0;\mp 1, -i, 0)/\sqrt{2}$

- Moduli squared of the helicity amplitudes [EPJ C59 (2009) 27]

$$\text{nonflip}(\lambda_W = \mp 1): |h^l_{\lambda_l=\mp\frac{1}{2}, \lambda_\nu=\pm\frac{1}{2}}|^2 = 8(q^2 - m_l^2),$$

$$\text{flip}(\lambda_W = 0): |h^l_{\lambda_l=\pm\frac{1}{2}, \lambda_\nu=\pm\frac{1}{2}}|^2 = 8\frac{m_l^2}{2q^2}(q^2 - m_l^2)$$

- Upper and lower signs refer to the configurations $(l^-, \bar{\nu}_l)$ $(\lambda_\nu = 1/2)$ and (l^+, ν_l) $(\lambda_\nu = -1/2)$, respectively
- In case of the e-mode only nonflip transition remains under assumption $\frac{m_e^2}{2q^2} \to 0$

Decay chain: $\frac{1}{2} \to \frac{1}{2} + \{0, \pm 1\}$ (2)

- Relation between helicity amplitudes and decay parameters

$$\text{normalization}\ \frac{1}{4}(1 \mp \cos\theta_l)^2 |H_{\frac{1}{2}1}|^2 + \frac{1}{4}(1 \pm \cos\theta_l)^2 |H_{-\frac{1}{2}-1}|^2 + \frac{1}{2}\sin^2\theta_l (|H_{-\frac{1}{2}0}|^2 + |H_{\frac{1}{2}0}|^2) = 1,$$

$$\alpha_D^{sl} = \frac{1}{4}(1 \mp \cos\theta_l)^2 |H_{\frac{1}{2}1}|^2 - \frac{1}{4}(1 \pm \cos\theta_l)^2 |H_{-\frac{1}{2}-1}|^2 + \frac{1}{2}\sin^2\theta_l (|H_{-\frac{1}{2}0}|^2 - |H_{\frac{1}{2}0}|^2),$$

$$\beta_D^{sl} = \frac{1}{\sqrt{2}} \sin\theta_l \sin\chi ((1 \pm \cos\theta_l)\Im(H^*_{-\frac{1}{2}0} H_{-\frac{1}{2}-1}) + (1 \mp \cos\theta_l)\Im(H^*_{\frac{1}{2}1} H_{\frac{1}{2}0})),$$

$$\gamma_D^{sl} = \frac{1}{\sqrt{2}} \sin\theta_l \cos\chi ((1 \pm \cos\theta_l)\Re(H^*_{-\frac{1}{2}0} H_{-\frac{1}{2}-1}) + (1 \mp \cos\theta_l)\Re(H^*_{\frac{1}{2}1} H_{\frac{1}{2}0}))$$

where $\beta_D^{sl} = \sqrt{1 - (\alpha_D^{sl})^2} \sin\phi_D^{sl}$ and $\gamma_D^{sl} = \sqrt{1 - (\alpha_D^{sl})^2} \cos\phi_D^{sl}$

- Non-zero elements of the decay matrix $b_{\mu\nu}$

$$\begin{aligned}
b_{00} &= 1, & b_{21} &= \mp\gamma_D^{sl}\cos\theta\sin\phi \mp \beta_D^{sl}\cos\phi,\\
b_{03} &= \alpha_D^{sl}, & b_{22} &= \pm\beta_D^{sl}\cos\theta\sin\phi \mp \gamma_D^{sl}\cos\phi,\\
b_{10} &= \alpha_D^{sl}\cos\phi\sin\theta, & b_{23} &= \sin\theta\sin\phi,\\
b_{11} &= \mp\gamma_D^{sl}\cos\theta\cos\phi \pm \beta_D^{sl}\sin\phi, & b_{30} &= \alpha_D^{sl}\cos\theta,\\
b_{12} &= \pm\beta_D^{sl}\cos\theta\cos\phi \pm \gamma_D^{sl}\sin\phi, & b_{31} &= \pm\gamma_D^{sl}\sin\theta,\\
b_{13} &= \sin\theta\cos\phi, & b_{32} &= \mp\beta_D^{sl}\sin\theta,\\
b_{20} &= \alpha_D^{sl}\sin\theta\sin\phi, & b_{33} &= \cos\theta
\end{aligned}$$

- Each element of $b_{\mu\nu}$ is multiplied by $q^2 p$ where $p = \sqrt{M_+ M_-}/(2M_1)$
- Main parameters: α_D^{sl}, ϕ_D^{sl}

Semileptonic Λ decay

- Momenta and masses: $\Lambda(p_1, M_1) \to p(p_2, M_2) + e^-(p_e, m_e) + \bar{\nu}_e(p_{\bar{\nu}_e}, 0)$
- Standard set of invariant FF for the weak current-induced baryonic $1/2^+ \to 1/2^+$ transitions:

$$M_\mu = M_\mu^V + M_\mu^A = \langle p(p_2)|J_\mu^{V+A}|\Lambda(p_1)\rangle =$$

$$= \bar{u}(p_2)\left[\gamma_\mu\left(F_1^V(q^2) + F_1^A(q^2)\gamma_5\right) + \frac{i\sigma_{\mu\nu}q^\nu}{M_1}\left(F_2^V(q^2) + F_2^A(q^2)\gamma_5\right) + \frac{q_\mu}{M_1}\left(F_3^V(q^2) + F_3^A(q^2)\gamma_5\right)\right]u(p_1)$$

where $q_\mu = (p_1 - p_2)_\mu$

- For $\Lambda \to pe^-\bar{\nu}_e$ at $\mathcal{O}(\frac{m_e^2}{2q^2}) \to 0 \Rightarrow F_3^{V,A} \to 0$

Semileptonic Λ decay

- Momenta and masses: $\Lambda(p_1, M_1) \to p(p_2, M_2) + e^-(p_e, m_e) + \bar{\nu}_e(p_{\bar{\nu}_e}, 0)$
- Standard set of invariant FF for the weak current-induced baryonic $1/2^+ \to 1/2^+$ transitions:

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$$= \bar{u}(p_2)\left[\gamma_\mu\left(F_1^V(q^2) + F_1^A(q^2)\gamma_5\right) + \frac{i\sigma_{\mu\nu}q^\nu}{M_1}\left(F_2^V(q^2) + F_2^A(q^2)\gamma_5\right) + \frac{q_\mu}{M_1}\left(F_3^V(q^2) + F_3^A(q^2)\gamma_5\right)\right]u(p_1)$$

where $q_\mu = (p_1 - p_2)_\mu$

- For $\Lambda \to pe^-\bar{\nu}_e$ at $\mathcal{O}(\frac{m_e^2}{2q^2}) \to 0 \Rightarrow F_3^{V,A} \to 0$
- $\Lambda \to pe^-\bar{\nu}_e \Rightarrow \Lambda \to pW^-(\to e^-\bar{\nu}_e)$
- Helicity amplitude is $H_{\lambda_2\lambda_W} = (H^V_{\lambda_2\lambda_W} + H^A_{\lambda_2\lambda_W})$ with $(\lambda_2 = \pm 1/2;\ \lambda_W = 0, \pm 1)$:

$$\text{vector}\begin{cases} H^V_{\frac{1}{2}1} = \sqrt{2M_-}\left(-F_1^V - \dfrac{M_1+M_2}{M_1}F_2^V\right), \\ H^V_{\frac{1}{2}0} = \dfrac{\sqrt{M_-}}{\sqrt{q^2}}\left((M_1+M_2)F_1^V + \dfrac{q^2}{M_1}F_2^V\right), \end{cases} \qquad \text{axial-vector}\begin{cases} H^A_{\frac{1}{2}1} = \sqrt{2M_+}\left(F_1^A - \dfrac{M_1-M_2}{M_1}F_2^A\right), \\ H^A_{\frac{1}{2}0} = \dfrac{\sqrt{M_+}}{\sqrt{q^2}}\left(-(M_1-M_2)F_1^A + \dfrac{q^2}{M_1}F_2^A\right). \end{cases}$$

where $M_\pm = (M_1 \pm M_2)^2 - q^2$; $\quad H^{V,A}_{-\lambda_2,-\lambda_W} = \pm H^{V,A}_{\lambda_2\lambda_W}$

Joint angular distribution

- Process $e^+e^- \to J/\psi \to \Lambda\bar{\Lambda} \to pe^-\bar{\nu}_e\bar{p}\pi^+$

$$\mathrm{Tr}\rho_{pW\bar{p}} \propto \mathcal{W}(\boldsymbol{\xi};\boldsymbol{\omega}) = \sum_{\mu,\bar{\nu}=0}^{3} C_{\mu\bar{\nu}} b^{\Lambda}_{\mu 0} a^{\bar{\Lambda}}_{\bar{\nu}0}$$

  - $C_{\mu\bar{\nu}}(\theta_\Lambda; \alpha_\psi, \Delta\Phi)$
  - $b_{\mu 0}$ matrices for $1/2 \to 1/2 + \{0, \pm 1\}$ decays $\Leftrightarrow b^{\Lambda}_{\mu 0} \equiv b_{\mu 0}(\theta_p, \varphi_p, \theta_e, \chi, q^2; \alpha^{sl}_{\Lambda})$
  - $a_{\bar{\nu}0}$ matrices for $1/2 \to 1/2 + 0$ decays $\Leftrightarrow a^{\bar{\Lambda}}_{\bar{\nu}0} \equiv a_{\bar{\nu}0}(\theta_{\bar{p}}, \varphi_{\bar{p}}; \alpha_{\bar{\Lambda}})$
    - $\boldsymbol{\xi} \equiv (\theta_\Lambda, \theta_p, \varphi_p, \theta_e, q^2, \chi, \theta_{\bar{p}}, \varphi_{\bar{p}})$
    - $\boldsymbol{\omega} \equiv (\alpha_\psi, \Delta\Phi, \alpha^{sl}_{\Lambda}, \alpha_{\bar{\Lambda}})$

- Elements of decay matrix $b_{\mu 0}$ and $a_{\bar{\nu}0}$

$$\left.\begin{aligned} b_{00} &= 1, \\ b_{10} &= \alpha^{sl}_D \cos\phi \sin\theta, \\ b_{20} &= \alpha^{sl}_D \sin\theta \sin\phi, \\ b_{30} &= \alpha^{sl}_D \cos\theta \end{aligned}\right\rangle \Lambda \to pe^-\bar{\nu}_e \text{ where } \alpha^{sl}_D \equiv \alpha^{sl}_\Lambda \qquad \left.\begin{aligned} a_{00} &= 1, \\ a_{10} &= \alpha_D \cos\varphi \sin\theta, \\ a_{20} &= \alpha_D \sin\theta \sin\varphi, \\ a_{30} &= \alpha_D \cos\theta \end{aligned}\right\rangle \bar{\Lambda} \to \bar{p}\pi^+ \text{ where } \alpha_D \equiv \alpha_{\bar{\Lambda}}$$

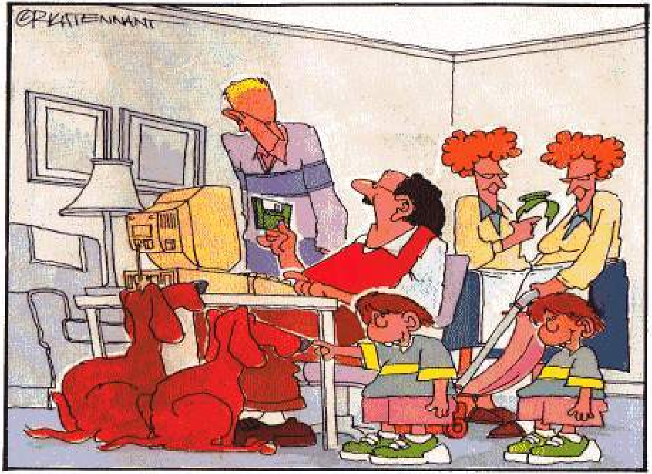
*each $b_{\mu 0}$ are multiplied by $q^2 p$ where $p = \sqrt{M_+ M_-}/(2M_1)$

[PRD 99 (2019) 056008]

Next steps

- Verify the correctness of the modular method for the semileptonic hyperon decays
  - Plan to use Andrzej's method (some package of scripts)
- Check the direction of particles in the helicity frame of the initial hyperon
- Expand the modular method to the semileptonic cascade decays, as an example:
  - $\Xi \to \Lambda(\to p + \pi) + l + \nu_l$
  - $\Xi \to \Sigma(\to p + \pi) + l + \nu_l$
  - ...

Backups



" I ALWAYS BACK UP EVERYTHING."

Form factors

$$F_i^{V,A}(q^2) = F_i^{V,A}(0)\prod_{n=0}^{n_i}\frac{1}{1-\frac{q^2}{m_{V,A}^2+n\alpha'^{-1}}} \approx F_i^{V,A}(0)\left(1+q^2\sum_{n=0}^{n_i}\frac{1}{m_{V,A}^2+n\alpha'^{-1}}\right)$$

	$F_i^{V,A}(0)$	$m_{V,A}$	α' [GeV^{-2}]	n_i
$F_1^V(q^2)$	$-\sqrt{\frac{3}{2}}$ [1]	$m_{K^*(892)^0}$ $(J^P=1^-)$		$n_1=1$
$F_2^V(q^2)$	$\frac{M_\Lambda \mu_p}{2M_p}F_1^V(0)$ [2]			$n_2=2$
$F_3^V(q^2)$	0 [4]	-	0.9	$n_3=2$
$F_1^A(q^2)$	$0.719F_1^V(0)$ [3]	$m_{K^*(1270)^0}$ $(J^P=1^+)$		$n_1=1$
$F_2^A(q^2)$	0 [4]	–		$n_2=2$
$F_3^A(q^2)$	$\frac{M_\Lambda(M_\Lambda+M_p)}{(m_{K^-})^2}F_1^A(0)$ [4]	m_K $(J^P=0^-)$		$n_3=2$

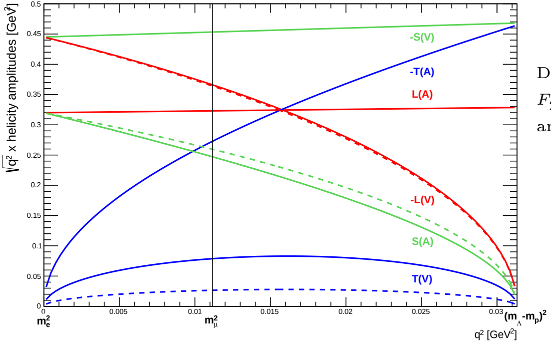
- [1] [PR135(1964)B1483], [PRL13(1964)264]
- [2] $\mu_p = 1.793$ [Lect.NotesPhys.222(1985)1], [Ann.Rev.Nucl.Part.Sci.53(2003)39], [JHEP0807(2008)132]
- [3] [PRD41(1990)780]
- [4] Vanish in the $SU(3)$ symmetry limit; Goldberger-Treiman relation [PR110(1958)1178], [PR111(1958)354]

Size estimations of helicity amplitudes

$\mathrm{T(V,A)} := \sqrt{q^2} H^{V,A}_{\frac{1}{2}1}$

$\mathrm{L(V,A)} := \sqrt{q^2} H^{V,A}_{\frac{1}{2}0}$

$\mathrm{S(V,A)} := \sqrt{q^2} H^{V,A}_{\frac{1}{2}t}$



Dashed lines: $F_2^V(q^2)$ and $F_3^A(q^2)$ are switched off

- If $q^2 = m_e^2 \Rightarrow H^V_{\frac{1}{2}0}$ and $H^A_{\frac{1}{2}0}$ are dominated
- If $q^2 = (M_\Lambda - M_p)^2 \Rightarrow H^A_{\frac{1}{2}1} = -\sqrt{2} H^A_{\frac{1}{2}0}$ are dominated
- Using data of the E-555 experiment (Fermilab) [PRD41 (1990) 780]
  $\Rightarrow \frac{F_1^A(0)}{F_1^V(0)}(\Lambda \to p e^- \bar{\nu}_e) = 0.731 \pm 0.016$ and $\frac{F_2^V(0)}{F_1^V(0)}(\Lambda \to p e^- \bar{\nu}_e) = 0.15 \pm 0.30$
  $\Rightarrow \frac{F_1^A(0)}{F_1^V(0)}(\Lambda \to p e^- \bar{\nu}_e) = 0.719 \pm 0.016$ with constraint $\frac{F_2^V(0)}{F_1^V(0)}(\Lambda \to p e^- \bar{\nu}_e) \to 0.97$ (CVC)

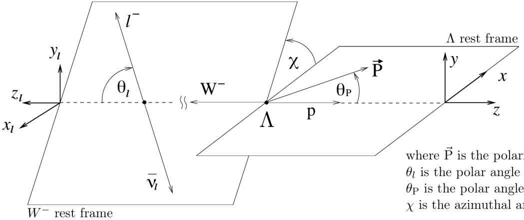
Semileptonic Λ decay (2)

- Angular distributions for the $\Lambda \to pW^-(\to e^-\bar{\nu}_e)$ of a polarized Λ in the helicity formalism [EPJ C59 (2009) 27]

$$\frac{d\Gamma}{dq^2 d\cos\theta_l d\chi d\cos\theta_{\rm P}} = \frac{1}{6}\frac{G_F^2}{(2\pi)^4}|V_{us}|^2\frac{q^2 p}{8M_\Lambda^2}\times$$
$$\times\left[\frac{3}{8}(1-\cos\theta_l)^2|H_{\frac12 1}|^2\rho_{-\frac12-\frac12}+\frac{3}{8}(1+\cos\theta_l)^2|H_{-\frac12-1}|^2\rho_{\frac12\frac12}+\frac{3}{4}\sin^2\theta_l(|H_{-\frac12 0}|^2\rho_{-\frac12-\frac12}+|H_{\frac12 0}|^2\rho_{\frac12\frac12})+\right.$$
$$\left.+\frac{3}{2\sqrt{2}}\rho_{\frac12-\frac12}\cos\chi\sin\theta_l\Big(H_{-\frac12 0}H_{-\frac12-1}(1+\cos\theta_l)+H_{\frac12 0}H_{\frac12 1}(1-\cos\theta_l)\Big)\right]$$

where spin density matrix

$$\boxed{\rho_{\lambda_p-\lambda_W,\lambda_p-\lambda'_W}(\underline{\underline{\theta_{\rm P}}})} = \frac{1}{2}\begin{pmatrix}1+{\rm P}\cos\theta_{\rm P} & {\rm P}\sin\theta_{\rm P}\\ {\rm P}\sin\theta_{\rm P} & 1-{\rm P}\cos\theta_{\rm P}\end{pmatrix} \Rightarrow \boxed{\rho_{\lambda_p-\lambda_W,\lambda_p-\lambda'_W}} = \frac{1}{2}\begin{pmatrix}I_0+I_z & I_x-iI_y\\ I_x+iI_y & I_0-I_z\end{pmatrix}$$



where $\vec{\rm P}$ is the polarization vector of the Λ
θ_l is the polar angle of the lepton in the W^- decay plane
$\theta_{\rm P}$ is the polar angle of the Λ polarization vector in the Λ decay plane
χ is the azimuthal angle between two decay planes

MC generator (step 1)

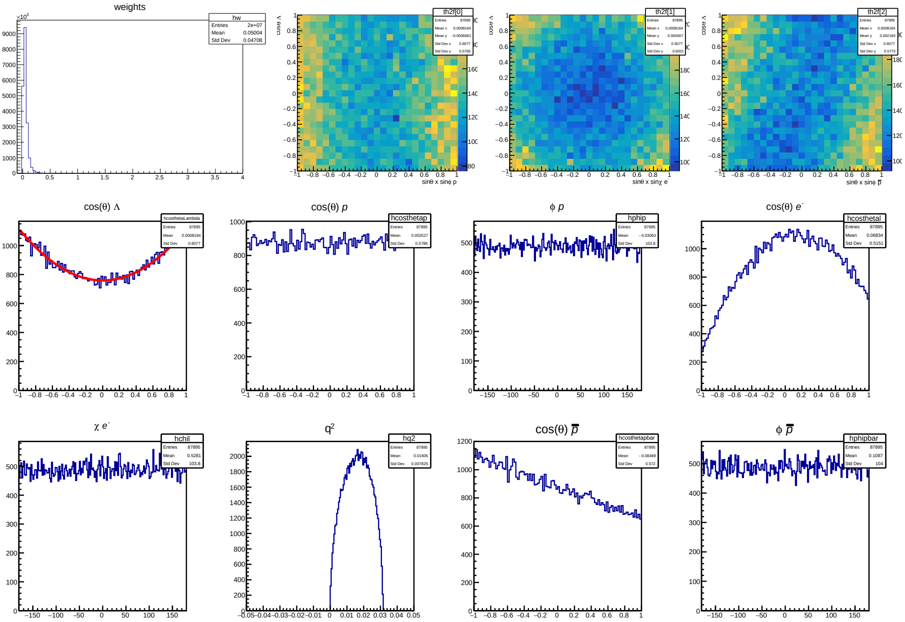
- Using `YYbar_example` package by Patrik, the presented process can be generated
  - Generate $N_{\text{evt}}^{\text{sig}}=10^6$ and $N_{\text{evt}}^{\text{phsp}}=10^7$
  - Generate $q^2 \in (m_e^2, (M_\Lambda - M_p)^2)$

```
pp[0] =  0.461;      // alpha_J/psi (arxiv:1808.08917)
pp[1] =  0.74;       // Delta_Phi (arxiv:1808.08917, equal 42.4deg)
pp[2] = 0.719;       // g_av=FA1_0/FV1_0 Lambda->p e- nu_ebar
pp[3] = 1.066;       // g_w=FV2_0/FV1_0 Lambda->p e- nu_ebar
pp[4] = -0.758;       // alpha_D LambdaBar->pbar pi+ (arxiv:1808.08917)
```

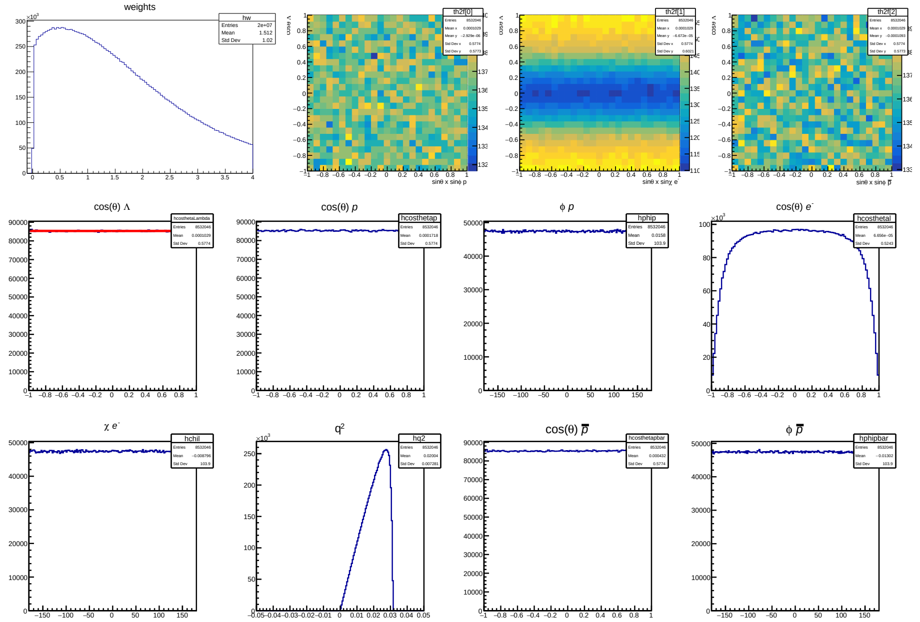
- No negative weights

```
Min: 1.09107e-06 Max:5.72174
nr of negative weights are: 0
 FCN=91.6295 FROM MIGRAD    STATUS=CONVERGED     104 CALLS         105 TOTAL
                     EDM=1.00386e-07    STRATEGY= 1      ERROR MATRIX ACCURATE
  EXT PARAMETER                                STEP         FIRST
  NO.   NAME      VALUE            ERROR          SIZE      DERIVATIVE
   1  p0           7.59231e+02   4.25138e+00   1.20418e-02  -4.15449e-05
   2  p1           4.69479e-01   1.55037e-02   4.39105e-05  -3.71647e-02
```

Random samples (signal) (step 1)



Random samples (phsp) (step 1)



Run through fit method (step 2)

- Set starting values in the fit

```
void mainMLL(){

      // instantiating the values to be measured
      Double_t pp[4];
      for( int i = 0; i < 4; i++ ) pp[i]=0;

      // starting values for fit
      Double_t alpha_jpsi      = 0.461;     // alpha_J/Psi
      Double_t dphi_jpsi       = 0.74;       // relative phase, Dphi_J/Psi
      Double_t gav_lam_plnu    = 0.719;      // FA1_0/FV1_0 (Lam->p l nubar_l)
      Double_t gv_lam_plnu     = 1.066;      // FV2_0/FV1_0 (Lam->p l nubar_l)
      Double_t alpha_lam_pbarpip  = -0.758;    // alpha (Lambar->pbar pi+)

      alpha_jpsi      = gRandom->Rndm();
      dphi_jpsi       = gRandom->Rndm();
      gav_lam_plnu      = 0.719;
      gv_lam_plnu     = 1.066;
      alpha_lam_pbarpip  = -0.758;

      ReadData();
      ReadMC();
```

- Fit method output

```
Loglike: -19457.7
 FCN=-19458.2 FROM MINOS     STATUS=SUCCESSFUL    254 CALLS         409 TOTAL
                     EDM=8.92012e-06    STRATEGY= 1      ERROR MATRIX ACCURATE
  EXT PARAMETER                PARABOLIC         MINOS ERRORS
  NO.   NAME      VALUE            ERROR      NEGATIVE      POSITIVE
   1  alpha_jpsi   4.58133e-01   9.71892e-03  -9.67526e-03   9.76061e-03
   2  dphi_jpsi    7.07653e-01   1.92031e-02  -1.90374e-02   1.93709e-02
   3  gaLam        9.83858e-01   1.21829e-02  -1.20563e-02   1.23126e-02
   4  gvLam        1.30270e+00   4.44752e-02  -4.44779e-02   4.44760e-02
   5  aLambar     -7.59457e-01   8.10586e-03  -8.07343e-03   8.13534e-03
                               ERR DEF= 0.5
 EXTERNAL ERROR MATRIX.    NDIM=  25    NPAR=  5    ERR DEF=0.5
  9.446e-05  3.862e-05  4.788e-06  1.294e-05  2.458e-05
  3.862e-05  3.688e-04 -1.519e-05 -3.984e-05  2.687e-05
  4.788e-06 -1.519e-05  1.484e-04  6.200e-06 -7.175e-08
  1.294e-05 -3.984e-05  6.200e-06  1.978e-03  2.434e-06
  2.458e-05  2.687e-05 -7.175e-08  2.434e-06  6.571e-05
 PARAMETER  CORRELATION COEFFICIENTS
       NO.  GLOBAL      1      2      3      4      5
        1  0.35404   1.000  0.207  0.040  0.030  0.312
        2  0.25216   0.207  1.000 -0.065 -0.047  0.173
        3  0.08553   0.040 -0.065  1.000  0.011 -0.001
        4  0.06216   0.030 -0.047  0.011  1.000  0.007
        5  0.33109   0.312  0.173 -0.001  0.007  1.000
```

Fit results (step 2)

- $g_{av} = \frac{F_1^A(0)}{F_1^V(0)}$, $g_w = \frac{F_2^V(0)}{F_1^V(0)}$

Parameter	$N_{sig}^{MC} = 10^6$ $N_{phsp}^{MC} = 10^7$	$N_{sig}^{MC} = 10^5$ $N_{phsp}^{MC} = 10^6$
α_Ψ	0.4581±0.0097	0.4867±0.0196
$\Delta\Phi$	0.7077±0.0192	0.7252±0.0398
$\alpha_{\bar{\Lambda}}$	-0.7595±0.0081	-0.762±0.0163
g_{av}	0.9839±0.0122	0.9793±0.024
g_w	1.3027±0.0445	1.2793±0.0878

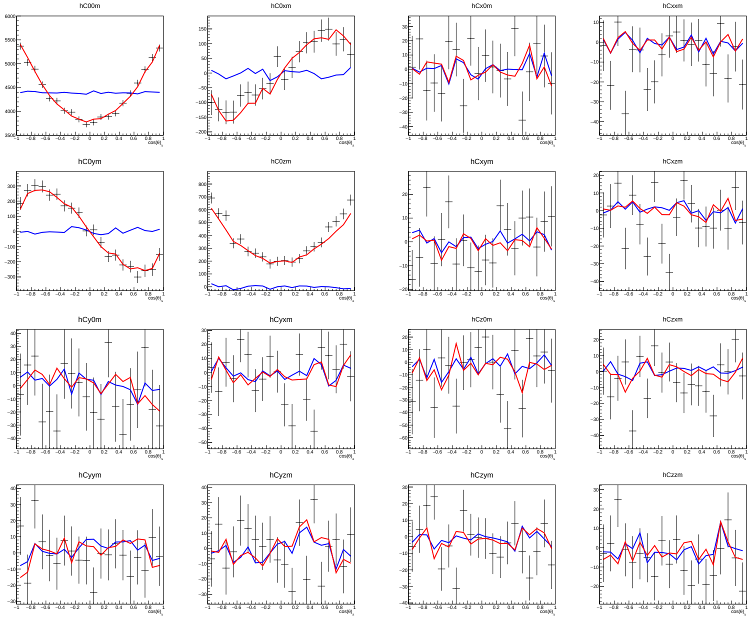
Parameter $N_{sig}^{MC} = 10^6$ $N_{phsp}^{MC} = 10^7$	e^--mode	μ^--mode
α_Ψ	0.4581±0.0097	0.475±0.003
$\Delta\Phi$	0.7077±0.0192	0.6886±0.0057
$\alpha_{\bar{\Lambda}}$	-0.7595±0.0081	-0.7509±0.0024
g_{av}	0.9839±0.0122	1.1456±0.0058
g_w	1.3027±0.0445	1.6243±0.0156

Fit method of moments (step 3) (1)

- Use fitted parameter values as input

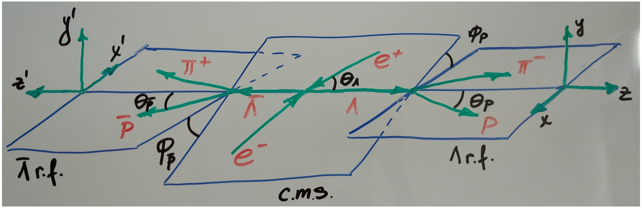
```
void momentsLLbar_semil(){

 Double_t pp[5];
   for( int i = 0; i < 5; i++ ) pp[i]=0.;

   // nominal
    pp[0] =  0.4581;     // alpha_J/psi
    pp[1] =  0.7077;     // Delta_Phi
    pp[2] =  0.9839;     // FA1_0/FV1_0 Lambda->p e- nu_ebar
    pp[3] =  1.3027;     // FV2_0/FV1_0 Lambda->p e- nu_ebar
    pp[4] = -0.7595;      // alpha_D LambdaBar->pbar pi+

 AngDisLLbar angdis(pp[0],pp[1],pp[2],pp[3],pp[4]);
```

Fit method of moments (step 3) (2)



Λ direction in c.m.s. (1)

- $e^+e^- \to \Lambda\bar{\Lambda} \to p\pi^-\bar{p}\pi^+$



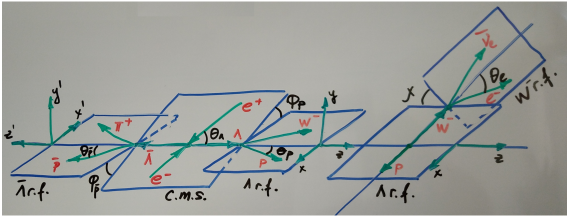
- $\Lambda \to p\pi^-$ (c.m.s.)
  - x: $\sin\theta_p \cos\phi_p$
  - y: $\sin\theta_p \sin\phi_p$
  - z: $\cos\theta_p$

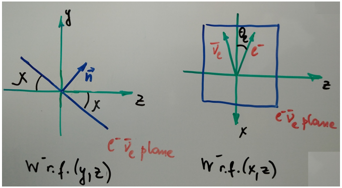
where $x = -x'$, $y = y'$, $z = -z'$

- $\bar{\Lambda} \to \bar{p}\pi^+$ (c.m.s.)
  - x': $\sin\theta_{\bar{p}} \cos\phi_{\bar{p}}$
  - y': $\sin\theta_{\bar{p}} \sin\phi_{\bar{p}}$
  - z': $\cos\theta_{\bar{p}}$

Λ direction in c.m.s. (2)

- $e^+e^- \to \Lambda\bar{\Lambda} \to pl^-\bar{\nu}_l\bar{p}\pi^+$





- W^- rest frame
  - x: -cos θ_l
  - y: sin θ_l sin χ
  - z: sin θ_l cos χ
- Λ rest frame
  - x: -cos θ_l sin θ_p
  - y: sin θ_l sin χ
  - z: sin θ_l cos χ cos θ_p
- $\Lambda \to pl^-\bar{\nu}_l$ (c.m.s.)
  - x: -cos θ_l sin θ_p cos ϕ_p
  - y: sin θ_l sin χ sin ϕ_p
  - z: sin θ_l cos χ cos θ_p cos ϕ_p

where $x = -x'$, $y = y'$, $z = -z'$

- $\bar{\Lambda} \to \bar{p}\pi^+$ (c.m.s.)
  - x': sin $\theta_{\bar{p}}$ cos $\phi_{\bar{p}}$
  - y': sin $\theta_{\bar{p}}$ sin $\phi_{\bar{p}}$
  - z': cos $\theta_{\bar{p}}$

$\Lambda\bar{\Lambda}$ direction in c.m.s. (3)

```
Double_t cosI(Int_t i,Int_t ip,Double_t z,Double_t ph)
{
  // calculate directional cos
  // i -> 0,x,y,z
  // ip ->1,2 //Lambda,Lambda_bar
  if(i==0) return 1;
  if(i==2) return TMath::Sqrt(1-z*z)*TMath::Sin(ph);
  Double_t val = 0;
  if(i==1) val = TMath::Sqrt(1-z*z)*TMath::Cos(ph);
  if(i==3) val = z;
  if(ip==2) return -val;
  if(ip==1) return val;
  return 0;

}
Double_t cosI_semil(Int_t i,Int_t ip,Double_t z,Double_t ph,Double_t zl,Double_t phl)
{
  // calculate directional cos
  // i -> 0,x,y,z
  // ip ->1,2 //Lambda,Lambda_bar
  if(i==0) return 1;
  if(i==2) return TMath::Sin(ph)*TMath::Sqrt(1-zl*zl)*TMath::Sin(phl);
  Double_t val = 0;
  if(i==1) val = -TMath::Sqrt(1-z*z)*TMath::Cos(ph)*zl;
  if(i==3) val = z*TMath::Cos(phl)*TMath::Sqrt(1-zl*zl);
  if(ip==2) return -val;
  if(ip==1) return val;
  return 0;

}
```

Some distributions (step 3)