Progress of Lattice QCD on Spectroscopy

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A few words in advance

- Thank the organizers for the invitation

- There are too many aspects about hadron spectrum
- I can only pick a few topics that I am more familiar with (and are also more relevant for HIEPA...)
- I will mainly focus on charm related stuff

- Thanks also go to my collaborators in CLQCD and elsewhere

Basic procedure of a lattice QCD calculation

QCD action in terms of basic fields ($\bar{\psi}$, ψ, U_μ):

$$S_{LQCD}[\bar{\psi}, \psi, U_\mu(x)] = S_g[U_\mu(x)] + \bar{\psi}_x \mathcal{M}[U_\mu(x)]_{xy}\psi_y \ , \qquad (1)$$

where $\mathcal{M}[U_\mu(x)]$ is called the fermion matrix, which differentiate different lattice fermions (staggered, Wilson, twisted mass, etc.). Any physical quantity (observable) $\mathcal{O}[\bar{\psi}, \psi, U_\mu]$ is given by

$$\begin{cases} \langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int DU_\mu D\bar{\psi} D\psi \mathcal{O}[\bar{\psi}, \psi, U_\mu] e^{-S_{LQCD}[\bar{\psi},\psi,U_\mu(x)]} \\ \mathcal{Z} = \int DU_\mu D\bar{\psi} D\psi e^{-S_{LQCD}} = \int DU_\mu e^{-S_g[U_\mu]} \det \mathcal{M}[U_\mu] \end{cases} \qquad (2)$$

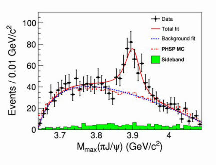
resembling an ensemble average with probability density:

$$\mathcal{P}[U_\mu] = \frac{1}{\mathcal{Z}} e^{-S_g[U_\mu]} \det \mathcal{M}[U_\mu] \ . \qquad (3)$$

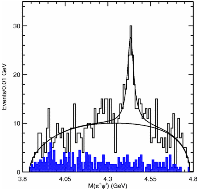
Omitting $\det \mathcal{M}$ yields the quenched approximation.

Outline

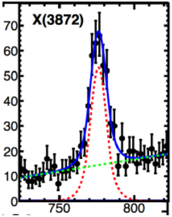
1. Charmed meson scattering and the XYZ's



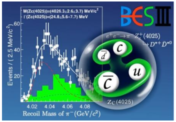
$Z_c(3900)$



$Z(4430)$



$X(3872)$



$Z_c(4025)$

- quarkonium-like states: valence quark structure $Q\bar{Q}q'\bar{q}$
- Neutral ones, $q = q'$, e.g. $X(3872)$, $Y(4260)$, etc.
- Charged ones, $q \neq q'$, $Z_c(3900)$, $Z_c(4025)$, $Z(4430)$, etc.
- Close to thresholds of mesons: $Q\bar{q}$ and $\bar{Q}q'$

Plus the newly discovered pentaquark states: P_c^+, etc.

Basic procedures in a lattice spectroscopy computation

The first step: GEVP
in a typical lattice spectrum calculation

- A set of interpolating operators with the "right" quantum numbers: $\{\mathcal{O}_\alpha : \alpha = 1, 2, \cdots, N_{op}\}$
- Compute (numerically via Monte Carlo) the correlation matrix:

$$\mathcal{C}_{\alpha\beta}(t,0) = \langle \mathcal{O}_\alpha(t)\mathcal{O}^\dagger_\beta(0)\rangle \ , \tag{4}$$

- Solve the so-called Generalized Eigen-Value Problem (GEVP) for the eigenvalues λ_α's,

$$\mathcal{C}(t,0)\cdot v_\alpha = \lambda_\alpha(t,t_0)\mathcal{C}(t_0,0)\cdot v_\alpha \ , \tag{5}$$

  for some appropriately chosen t_0
- From the eigenvalues $\lambda_\alpha(t,t_0)$, extract the corresponding eigenvalues of the Hamiltonian: E_α via

$$\lambda_\alpha(t,t_0) \sim e^{-E_\alpha(t-t_0)} \ . \tag{6}$$

- Pass the E_α's to the second step

Complications

1. E_α's are NOT hadron mass values!
   - E_α is the eigenvalue of the QCD Hamiltonian
     - (in a latticized finite box!)
   - Most hadrons are resonances
2. Many types of operators enter (operator mixing)!
   - single hadron operators
   - multi-hadron operators (esp. beyond the threshold)...

The second step

Relate the E_α's to the spectral quantities

0. E_α's are "approximate" hadron masses
   - only if the hadron is stable or the resonance is "narrow" enough
1. Using a version of the Lüscher formalism
   - single channel version has matured over the years
   - multi-channel applications appearing recently
   - more channels,rather complicated
2. Using other approaches
   - the effective Hamiltonian approach
   - the HALQCD approach
   - the optical potential approach

Lüscher's approach

in theory (e.g. M. Lüscher, NPB354, 531, 1991)

- original: single-channel spinless two-particle elastic scattering in COM frame,

$$E_\alpha(L) \Leftrightarrow \delta(E_\alpha)\ . \tag{7}$$

$$\begin{cases} \tan\delta(\bar{k}) = \dfrac{\pi^{3/2} q}{Z_{00}(1, q^2)}\ , \\ 2\sqrt{\bar{k}^2 + m^2} = E(L)\ , \quad q = kL/(2\pi)\ . \end{cases} \tag{8}$$

- extensions over the years
  - to particles with spin
  - to multi-channels
  - different BC's,
  - different frames,
  - ...

Other approaches: the HALQCD method

see e.t. N. Ishii et al, PRL99, 022001,2007; PLB712,437,2012.

- starts from the so-called NBS wavefunction (a four-point function)

$$F_{N\Omega}(\mathbf{x}-\mathbf{y}, t-t_0) = \langle 0|N_\alpha(\mathbf{x},t)\Omega_{\beta,l}(\mathbf{y},t)\bar{J}_{N\Omega}(t_0)|0\rangle \quad (9)$$

- the potential is obtained via the time-dependent HALQCD approach,

$$V_C(r) \simeq \frac{1}{2\mu}\nabla^2 R(r,t)/R(r,t) - \frac{\partial}{\partial t}\ln R(r,t) \;, \quad (10)$$

with $R(r,t) = F_{N\Omega}(r,t)/e^{-(m_N+m_\Omega)t}$.

☞ no need for GEVP

Lattice QCD studies on XYZ particles

- Investigate two-particle spectra only
  - This gives us a first impression
  - however, somewhat indirect
- Using Lüscher formalism (a relation from two-particle spectra to scattering matrix elements)
  - too many channels
  - although multi-channel Lüscher is known, it is rather cumbersome
  - single-channel approximation near threshold
  - however, what if this is not valid?
- Using HALQCD approach
  - defines a NBS wavefunction which is measurable on lattices
  - extract from NBS wavefunction a "potential"
  - compute the scattering properties from using extracted potential

Charmed meson near-threshold scattering

$N_f = 2$ twisted mass confs., using single-channel Lüscher

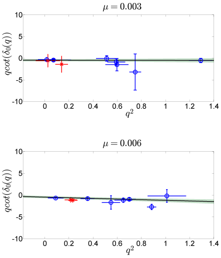
- $(D^* \bar{D}^*)^{\pm}$ ($Z_c(4025)$)

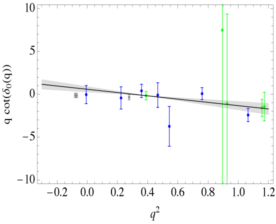
  CLQCD, PRD92 054507 (2015)

  $(D\bar{D}^*)^{\pm}$ ($Z_c(3900)$)

  CLQCD, PRD89 094506 (2014)

  - TBC utilized
  - 3 m_π values: 300,425,485MeV
  - weakly repulsive interaction found
  - no indication of a bound state





$$q \cot \delta(q^2) = \frac{1}{a_0} + \frac{1}{2} r_0 q^2 + \cdots ,$$

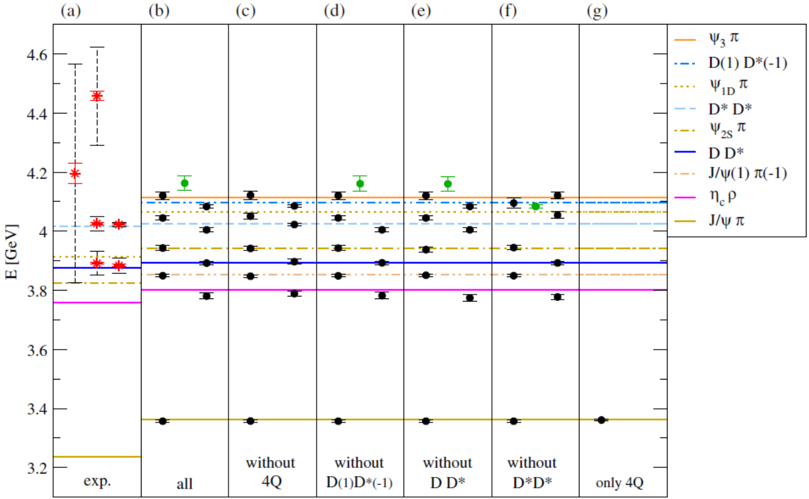
☞ need more ensembles
to inspect chiral & finite volume behavior

For $Z(4430)$

$N_f = 2$ twisted mass confs., using single-channel Lüscher

- $(\bar{D}_1 D^*)^{\pm}$ $(Z(4430))$ CLQCD, Phys.Rev. D93 (2016)
  - attractive interaction shows up
  - appears to be more attractive than the quenched results G. Meng et al, PRD80 034503 (2009)
  - some indications of a bound state seen
  - however, needs more volumes

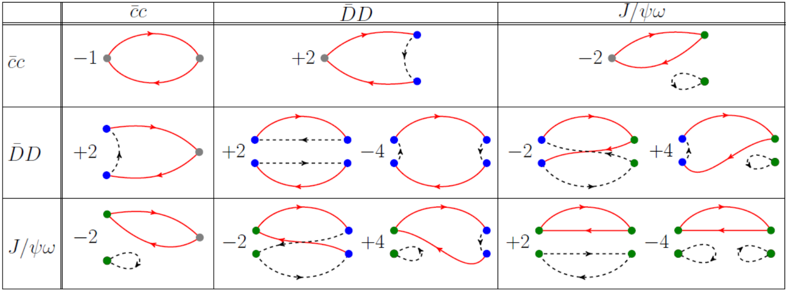
S. Prelovsek et al study on Z_c's etc.



- Study the spectrum and compare with the free

S. Prelovsek et al, PRD91 014504 2015; 1405.7623

Vector and scalar charmonium resonances with lattice QCD beyond the threshold



C.B. Lang et al JHEP 1509 (2015) 089;1503.05363

- $c\bar{c}$, $D\bar{D}$ and $J/\psi\omega$ operators in scalar channel
- $c\bar{c}$, $D\bar{D}$ operators in vector channel

- Assuming elastic scattering near threshold......

☞ $\psi(3770)$ well described by $D\bar{D}$ p-wave scattering

☞ scalar channel is still full of puzzles

X(3872) and Y(4140) using diquark-antidiquark operators

M. Padmanath et al, PRD92, 034501 (2015); 1503.03257

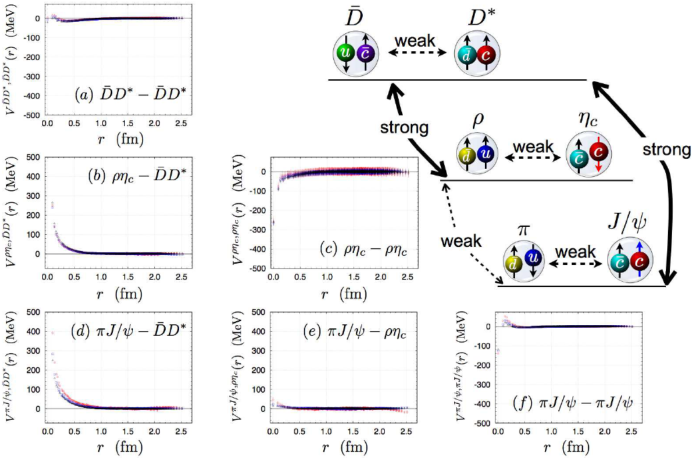
- use $\bar{c}c$ and $\bar{c}c(\bar{u}u + \bar{d}d)$ $\bar{c}c\bar{s}s$ type diquark-antidiquark operators in $J^{PC} = 1^{++}$ channel

☞ $X(3872)$ with $I = 0$ is observed

☞ $Y(4140)$ is not found

The HALQCD approach

$Z_c(3900)$, Phys. Rev. Lett. 117, 242001 (2016), 1602.03465



- $Z_c(3900)$ is NOT a conventional resonance
- appears to be a coupled channel effect arising from the $\pi J/\psi - \bar{D}D^* - \eta_c\rho$ coupling

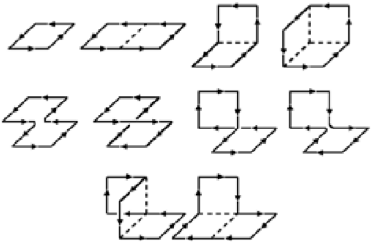
Summary of XYZ on the lattice

- The QCD nature of the XYZ states remains obscure
- More lattice studies are needed to clarify contraversies
  - more channels (two-channel study using Lüscher)
  - more volumes and pion mass values
  - comparison with different methods (HALQCD/Lüscher)
  - other methods?

- Glueballs are well-defined objects in non-abelian pure gauge theory
- In full QCD, glueballs mix with the conventional/unconventional hadrons with the same quantum numbers
- Pure $SU(3)$ gauge glueball spectra has been studied extensively on the lattice

Morningstar& M. Peardon, PRD 60, 034509, 1999; Chen et al, PRD 73, 014516, 2006

- operators are built from different Wilson loops

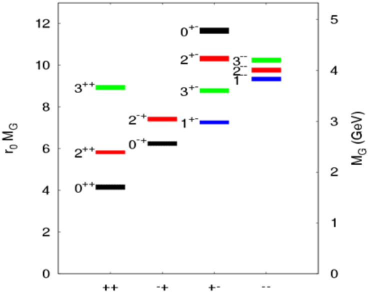


R \ J	0	1	2	3	4	5
A_1	1	0	0	0	1	0
A_2	0	0	0	1	0	0
E	0	0	1	0	1	1
T_1	0	1	0	1	1	2
T_2	0	0	1	1	1	1

Basic picture

- scalar glueball $\sim 1.5 - 1.7$GeV
- tensor glueball ~ 2.4GeV

- In real world, glueballs mix with conventional hadrons
- Scalar candidates: $f_0(1500)$, $f_0(1370)$, $f_0(1710)$...
- Tensor candidates: $f_2(2340)$,...
- What happened in full QCD with sea quarks?



Glueballs in unquenched QCD: exploratory

CLQCD, arXiv:1702.08174

- To address these questions, we have explored the glueball spectrum using $N_f = 2$ clover Wilson quarks on anisotropic lattices
- We have utilized only gluonic operators.
- Scalar, tensor and pseudoscalars are studied
  - Scalars: mixing seems to be serious with $q\bar{q}$ meson states
  - Tensors: more or less agrees with the quenched picture
  - Pseudoscalars: depends on how to probe it, using gluonic operators $\mathbf{B} \cdot (\nabla \times \mathbf{B})$ or the topological charge density $\mathbf{E} \cdot \mathbf{B}$

TABLE VII: We compare our results with previous results both from the quenched lattice QCD studies [8, 9] and the full-QCD study [17]. We average the masses of E^{++} and T_2^{++} states to obtain the estimate of the 2^{++} glueball mass.

	m_π (MeV)	$m_{0^{++}}$ (MeV)	$m_{2^{++}}$ (MeV)	$m_{0^{-+}}$ (MeV)
$N_f = 2$	938	1417(30)	2363(39)	2573(55)
	650	1498(58)	2384(67)	2585(65)
$N_f = 2+1$ [17]	360	1795(60)	2620(50)	—
quenched [8]	—	1710(50)(80)	2390(30)(120)	2560(35)(120)
quenched [9]	—	1730(50)(80)	2400(25)(120)	2590(40)(130)

Radiative decays of charmonium to glueballs

- To lowest order in QED, amplitude for $J/\psi \to \gamma H$ is given by

$$M_{r,r_\gamma,r_H} = \epsilon^*_\mu(\vec{q}, r_\gamma)\langle H(\vec{p}_f, r_H)|j^\mu(0)|J/\psi(\vec{p}_i, r)\rangle, \qquad (11)$$

  - $\vec{p}_i$: initial momentum of J/ψ
  - $\vec{p}_f$: final momentum of hadron H
  - $\vec{q} = \vec{p}_i - \vec{p}_f$: momentum of the real photon
  - r/r_γ: polarization of initial J/ψ/photon
  - r_H: polarization of final hadron (if needed)
  - $\epsilon(\vec{q}, r_\gamma)$: polarization vector of photon
  - $j^\mu(0)$: electromagnetic current operator

☞ Matrix element $\langle H(\vec{p}_f, r_H)|j^\mu(0)|J/\psi(\vec{p}_i, r)\rangle$ is non-perturbative in nature

☞ One of the precision tests for lattice in the case of $J/\psi \to \gamma\eta_c$, see e.g.C. Davies et al, 1301.7203

Three-point function

$$\Gamma^{(3)}_{i,\mu,j}(\vec{p}_f,\vec{q};t_f,t)=\frac{1}{T}\sum_{\vec{y},\tau=0}^{T-1}e^{-i\vec{q}\cdot\vec{y}}\langle\Phi^{(i)}(\vec{p}_f,t_f+\tau)J_\mu(\vec{y},t+\tau)O_{V,j}(\vec{0},\tau)\rangle \quad (12)$$

- $J_\mu=\bar{c}\gamma_\mu c$: vector current for the charm quark
- $O_{V,j}=\bar{c}\gamma_j c$: interpolating operator for J/ψ
- $\Phi^{(i)}$ interpolating operator for final hadron H

Multipole decomposition

$$\langle H(\vec{p}_f, r_H)|j^\mu(0)|J/\psi(\vec{p}_i, r)\rangle = \alpha_1^\mu E_1(Q^2) + \alpha_2^\mu M_2(Q^2) + \alpha_3^\mu E_3(Q^2) + \alpha_4^\mu C_1(Q^2) + \alpha_5^\mu C_2(Q^2) \quad (13)$$

where the first is for scalar while the second is for tensor glueball.

- α's are known functions of p_f and p_i;
- $Q^2 = -(p_i - p_f)^2$ is the photon four-momentum squared

☞ Physical photon point: $Q^2 = 0$

- $E_i(Q^2)$, $M_2(Q^2)$, $C_i(Q^2)$'s form factors
- Scalar: only $E_1(0)$ enters the decay rate
- Tensor: $E_1(0)$, $M_2(0)$ and $E_3(0)$ enter the decay rate

One can also calculate $J/\psi \to \gamma G$

quenched, see CLQCD, PRL110 021601 (2013); PRL111 091601 (2013)

- scalar glueball

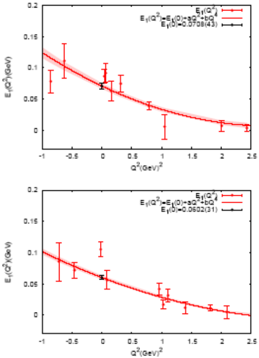


FIG. 2: The extracted form factors $E_1(Q^2)$ in the physical units. The upper panel is for $\beta = 2.4$ and the lower one for $\beta = 2.8$. The curves with error bands show the polynomial fit with $E_1(Q^2) = E_1(0) + aQ^2 + bQ^4$, as the black dot is the interpolated value $E_1(0)$ at $Q^2 = 0$.

- tensor glueball

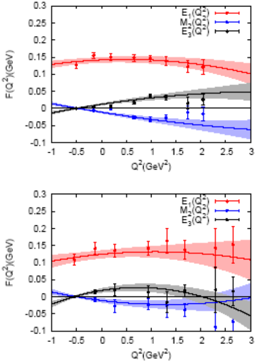


FIG. 2: The extracted form factors $E_1(Q^2)$ $M_2(Q^2)$ a $E_3(Q^2)$ in the physical units. The upper panel is for $\beta =$ and the lower one for $\beta = 2.8$. The curves with error bar show the polynomial fit with $F_i(Q^2) = F_i(0) + a_iQ^2 + b_iQ$

Comparison with experimental results: scalar

Lattice prediction:

$$\Gamma(J/\psi \to \gamma G_{0^+}) = \frac{4}{27}\alpha \frac{|p|}{M_{J/\psi}^2}|E_1(0)|^2 = 0.35(8) keV$$

$$\Gamma/\Gamma\_tot = 0.33(7)/93.2 = 3.8(9)\times 10^{-3}$$

C. Amsler et al. (Particle Data Group), Phy. Rev. D 86, 010001 (2012)

$J/\psi \to \gamma f_0(1500) \to \gamma\pi\pi \quad (1.01\pm 0.32)\times 10^{-4}$

$Br(f_0(1500)\to \pi\pi) = (34.9\pm 2.3)\% \quad \Rightarrow Br(J/\psi \to \gamma f_0(1500)) = 2.9\times 10^{-4}$

$J/\psi \to \gamma f_0(1710) \to \gamma\eta\eta$	$(2.35^{+1.27}_{-0.77})\times 10^{-4}$
$J/\psi \to \gamma f_0(1500) \to \gamma\eta\eta$	$(1.65^{+0.57}_{-1.50})\times 10^{-4}$

BESIII results (PRD87, 092009)

$J/\psi \to \gamma f_0(1710) \to \gamma K\bar{K}$	$(8.5^{+1.2}_{-0.9})\times 10^{-4}$
$J/\psi \to \gamma f_0(1710) \to \gamma\pi\pi$	$(4.0\pm 1.0)\times 10^{-4}$
$J/\psi \to \gamma f_0(1710) \to \gamma\omega\omega$	$(3.1\pm 1.0)\times 10^{-4}$
$J/\psi \to \gamma f_0(1710)$	$>(1.5\pm 0.3)\times 10^{-3}$

Using $Br(f_0(1710)\to K\underline{K})=0.36 \Rightarrow Br(J/\psi\to\gamma f_0(1710)) = 2.4\times 10^{-3}$

$Br(f_0(1710)\to \pi\pi)=0.15 \Rightarrow Br(J/\psi\to\gamma f_0(1710)) = 2.7\times 10^{-3}$

Our result support f0(1710) as the candidate for the scalar glueball

Comparison with experimental results: tensor

- our lattice results suggests a large branching ratio for $J/\psi \to \gamma G_{2^{++}}$, of about 10^{-2}.
- BESIII has results for $J/\psi \to \gamma\eta\eta$ and $J/\psi \to \gamma\phi\phi$

  Ablikim et al.,PRD87, 092009 (2013);Ablikim et al., PRD93, 112011 (2016)
- They identified a broad tensor object $f_2(2340)$
- Using these info, we estimate

$$Br(J/\psi \to \gamma f_2(2340)) \sim 10^{-2} \,. \tag{14}$$

This is consistent with our lattice estimate.

☞ it is possible that $f_2(2340)$ contains a large tensor glueball component

Summary and outlook

1. As a theoretical tool from first principles, Lattice QCD has become an important player in charm related physics
2. For the *XYZ* particles, especially $Z_c(3900)$...
   - Direct search in the two-particle spectra yields nothing
   - single channel scattering also finds no indications
   - HALQCD claims it is a couple-channel effect
   - ☞ more detailed study using Lüscher...
3. For the glueballs
   - Unquenched spectra look similar (using only gluonic ops.)
   - Needs to address mixing with $q\bar{q}$ operators
   - Our lattice results on $J/\psi \to \gamma G$ favors $f_0(1710)$ and $f_2(2340)$ as scalar and tensor glueball candidates, respectively

Thank you for your attention!