### W-exchange contribution to the decays $\Xi_{cc}^{++} \rightarrow \Xi_{c}^{+(\prime)}\pi^{+}$ using light-cone sum rule

#### Shi Yu-Ji 施瑀基 华东理工大学

2023年4月9日@合肥

Cooperated with Zhen-Xing Zhao, Ye Xing, and Ulf-G. Meissner Phys.Rev.D 106 (2022) 3, 034004

- Observation of doubly charmed baryon
- Status of the theoretical studies on  $\Xi_{cc}^{++} \rightarrow \Xi_{c}^{+(\prime)}\pi^{+}$ : W-emission and W-exchange contribution
- Light-cone sum rules calculation for the W-exchange diagram
- Numerical Results: Amplitudes and branching fraction

# Motivation

Observation of doubly charmed baryon

#### Prediction from the quark model







#### **Decuplet**

#### Searching for the doubly charmed baryon



#### Searching for the doubly charmed baryon



It is firstly predicted by: F.-S. Yu, H.-Y. Jiang, R.-H. Li, C.-D. Lü, W. Wang and Z.-X. Zhao, Chin. Phys. C 42, 051001 (2018)

In 2018, the LHCb collaboration observed a two-body decay of  $\Xi_{cc}^{++}$ : Phys. Rev. Lett. 121, 162002 (2018)

$$\Xi_{cc}^{++} \to \Xi_{c}^{+} \pi^{+} \qquad \mathcal{R}(\mathcal{B}) \equiv \frac{\mathcal{B}(\Xi_{cc}^{++} \to \Xi_{c}^{+} \pi^{+}) \times \mathcal{B}(\Xi_{c}^{+} \to pK^{-} \pi^{+})}{\mathcal{B}(\Xi_{cc}^{++} \to \Lambda_{c}^{+} K^{-} \pi^{+} \pi^{+}) \times \mathcal{B}(\Lambda_{c}^{+} \to pK^{-} \pi^{+})} = 0.035 \pm 0.009 (\text{stat}) \pm 0.003 (\text{syst})$$

Also predicted by: Chin. Phys. C 42, 051001 (2018)

In 2022, a similar two-body decay of  $\Xi_{cc}^{++}$  was observed by LHCb: JHEP 05 (2022) 038

$$\Xi_{cc}^{++} \to \Xi_{c}^{+\prime} \pi^{+} \qquad \frac{\mathcal{B}(\Xi_{cc}^{++} \to \Xi_{c}^{+\prime} \pi^{+})}{\mathcal{B}(\Xi_{cc}^{++} \to \Xi_{c}^{+} \pi^{+})} \equiv \frac{\mathcal{B}'}{\mathcal{B}} = 1.41 \pm 0.17 \pm 0.1$$

Not consistent with theoretical predictions

### The weak decay of $\Xi_{cc}^{++} \rightarrow \Xi_{c}^{+(\prime)} \pi^{+}$

 $\Xi_{cc}^{++} \rightarrow \Xi_{c}^{+(\prime)}\pi^{+}$  receives contributions from two topological diagrams: the W-emission diagram (left) and the W-exchange diagram (right):



Factorizable (fac)

Non-factorizable (nf)

#### **Recently, the theoretical prediction show that:**

				$\frown$	
	Method	$\mathcal{B}(\Xi_{cc}^{++}\to\Xi_c^+\pi^+)$	$\mathcal{B}(\Xi_{cc}^{++}\to\Xi_c^{+\prime}\pi^+)$	$\mathcal{B}'/\mathcal{B}$	
	LFQM+PM	0.69~%	4.65~%	6.74	Too small
fac + nf:	3LCQM	0.71~%	3.39%	4.77	or too large
	HQET+PM	6.64~%	5.39~%	0.81	/
	NRQM+PM	9.19%	7.34~%	0.8	8

#### The W-emission diagram



$$\langle \Xi_{c}^{+(\prime)}(p-q)\pi^{+}(q)|\mathcal{H}_{\rm eff}(0)|\Xi_{cc}^{++}(p)\rangle_{\rm fac,nf} = i \ \bar{u}(p-q) \big[ A^{(\prime)\rm fac,nf} + B^{(\prime)\rm fac,nf}\gamma_{5} \big] u(p)$$

$\Xi_{cc}^{++} \to \Xi_c^{+(\prime)} \pi^+$	$A^{\mathrm{fac}}$	$B^{\mathrm{fac}}$	$A'^{ m fac}$	$B'^{ m fac}$
QCDSR	-8.74	16.76	-3.55	34.13
m LFQM	7.40	15.06	4.49	48.50
3LQM	-8.13	-12.97	-4.34	-37.59
NRQM	7.38	16.77	4.29	53.65
HQET	9.52	19.45	5.10	62.37

QCDSR: Y. J. Shi, W. Wang and Z. X. Zhao, Eur. Phys. J. C 80, no.6, 568 (2020)

- LFQM: H. Y. Cheng, G. Meng, F. Xu and J. Zou, Phys. Rev. D 101, no.3, 034034 (2020)
- 3LQM: T. Gutsche et al. Phys. Rev. D 99, no.5, 056013 (2019)

NRQM: HQET: R. Dhir and N. Sharma, Eur. Phys. J. C 78, no.9, 743 (2018)

#### The W-exchange diagram



H. Y. Cheng, G. Meng, F. Xu and J. Zou, Phys. Rev. D 101, no.3, 034034 (2020)

# Theoretical Method

### Light-cone sum rules

Proposed by: A. Khodjamirian for  $B \rightarrow \pi\pi$ : Nucl. Phys. B 605, 558-578 (2001)

#### The framework of light-cone sum rules



#### Hadron Level: insert $\Xi_c$



$$\Pi_{H}^{\mathcal{O}_{i}}(p,q,k)_{\mathrm{WE}} = i^{3} \int d^{4}x \ e^{-i(p-q)\cdot x} \sum_{\pm',\sigma'} \frac{1}{(p-k)^{2} - m_{\Xi_{c}}^{\pm'/2}} \times \lambda_{\Xi_{c}}^{\pm'} u^{\pm'}(p-k,\sigma') \langle p-k,\sigma',\pm'|\mathcal{O}_{i}(0)\bar{J}_{\Xi_{cc}}(x)|\pi^{-}(q)\rangle + \int_{s_{\Xi_{c}}}^{\infty} ds' \frac{\rho_{\Xi_{c}}(s',(p-q)^{2},P^{2})}{s'-(p-k)^{2}}, \qquad \text{Only depends on}$$
Excited states and  $(p-q)^{2}$  and  $P^{2} = (p-k-q)^{2}$ ,  $p^{2} = k^{2} = 0$   $q^{2} = m_{\pi}^{2} \approx 0.$ 





#### **Quark-Hadron duality**

$$\Pi_{H}^{\mathcal{O}_{i}}(p,q,k)_{\mathrm{WE}} = \Pi_{QCD}^{\mathcal{O}_{i}}(p,q,k)_{\mathrm{WE}} = \frac{1}{\pi} \int_{(m_{c}+m_{s})^{2}}^{s_{\Xi_{c}}} ds' \frac{\mathrm{Im}\Pi_{QCD}^{\mathcal{O}_{i}}(s',(p-q)^{2},P^{2})_{\mathrm{WE}}}{s'-(p-k)^{2}} + \frac{1}{\pi} \int_{s_{\Xi_{c}}}^{\infty} ds' \frac{\mathrm{Im}\Pi_{QCD}^{\mathcal{O}_{i}}(s',(p-q)^{2},P^{2})_{\mathrm{WE}}}{s'-(p-k)^{2}} \int_{s_{\Xi_{c}}}^{\infty} ds' \frac{\rho_{\Xi_{c}}(s',(p-q)^{2},P^{2})}{s'-(p-k)^{2}} \mathbf{C} \text{anceled above } \mathbf{S}_{\Xi_{c}} - \mathbf{S}_{\Xi_{c}} > (m_{c} + m_{s})^{2}$$

**Borel Transformation for**  $(p - k)^2$ 

$$\mathcal{B}_{T'^2}\left[(p-k)^{2n}\right] = 0$$
  
$$\mathcal{B}_{T'^2}\left[\frac{1}{[s'-(p-k)^2]^n}\right] = \frac{1}{(n-1)!} \frac{\exp[-s'/T'^2]}{(p-k)^{2(n-1)}}$$
  
Excited states (large s') can be suppressed

We have to move the pion to the final state



The sum rules equation:

$$\sum_{\pm',\pm,\sigma',\sigma} e^{-m_{\Xi_{c}}^{\pm'2}/T'^{2} - m_{\Xi_{c}c}^{\pm2}/T^{2}} \lambda_{\Xi_{c}}^{\pm'} \lambda_{\Xi_{cc}}^{\pm} \times u^{\pm'}(p-k,\sigma') \langle p-k,\sigma',\pm';\pi^{+}(-q)|\mathcal{O}_{i}(0)|p-q,\sigma,\pm\rangle_{\rm WE} | \bar{u}^{\pm}(p-q,\sigma) = \frac{1}{\pi^{2}} \int_{(m_{c}+m_{s})^{2}}^{s_{\Xi_{c}}} ds \; e^{-s'/T'^{2}} e^{-s/T^{2}} {\rm Im}^{2} \Pi_{QCD}^{\mathcal{O}_{i}}(s',s,P^{2}).$$
Only depends on  $P^{2}$ 
Parameterizaed as:
$$i \; \bar{u}^{\pm'}(p-k,\sigma') \left[ A_{1,i}^{\pm'\pm}(P^{2}) + B_{1,i}^{\pm'\pm}(P^{2})\gamma_{5} + A_{2,i}^{\pm'\pm}(P^{2}) \frac{q}{m_{\Xi_{cc}}^{\pm}} + B_{2,i}^{\pm'\pm}(P^{2}) \frac{q\gamma_{5}}{m_{\Xi_{cc}}^{\pm}} \right] u^{\pm}(p-q,\sigma)$$

Two extra terms due to the non-vanishing k

#### Quark-Gluon Level



#### Quark-Gluon Level: Double imaginary part



## Numerical Results

#### Numerical Result: Borel parameters

$$\begin{split} i & \sum_{\pm \pm'} e^{-m_{\Xi_c}^{\pm'^2/T'^2} - m_{\Xi_{cc}}^{\pm 2}/T^2} \lambda_{\Xi_c}^{\pm'} \lambda_{\Xi_c}^{\pm} (\not\!\!\!\!/ p_2 + m_{\Xi_c}^{\pm'}) \left[ A_{1,i}^{\pm'\pm} + B_{1,i}^{\pm'\pm} \gamma_5 + A_{2,i}^{\pm'\pm} \frac{\not\!\!\!/}{m_{\Xi_{cc}}^{\pm}} + B_{2,i}^{\pm'\pm} \frac{\not\!\!/ p_3}{m_{\Xi_{cc}}^{\pm}} \right] (\not\!\!\!\!/ p_1 + m_{\Xi_{cc}}) \\ = & \frac{1}{\pi^2} \int_{(m_c + m_s)^2}^{s_{\Xi_c}} ds' \int_{4m_c^2}^{s_{\Xi_{cc}}} ds \ e^{-s'/T'^2} e^{-s/T^2} \mathrm{Im}^2 \Pi_{QCD}^{\mathcal{O}_i} (s', s, P^2)_{\mathrm{WE}}, \end{split}$$

#### The choice of Borel parameter T, T' must satisfy:



#### **Two assumptions :**

(1)  $\frac{T^2}{T'^2} \approx \frac{M_1^2 - m_1^2}{M_2^2 - m_2}$ 

 $M_{1(2)}$  is the mass of the initial (final) baryon and  $m_{1(2)}$  is the mass of the quark before(after) the weak decay.

Used in the study of *D* meson decays

P. Ball, V. M. Braun and H. G. Dosch, Phys. Rev. D 44, 3567-3581 (1991)



Blue band: uncertainty of the  $s_{\Xi_{cc}}$  and  $s_{\Xi_{c}}$ 

**Red band: uncertainty of MC integration** 



$$6 < T^{2} < 8 \text{ GeV}^{2} \text{ for } \Xi_{cc}^{++} \rightarrow \Xi_{c}^{+} \pi^{+}$$

$$5 < T^{2} < 7 \text{ GeV}^{2} \text{ for } \Xi_{cc}^{++} \rightarrow \Xi_{c}^{+\prime} \pi^{+}$$

$$s_{\Xi_{cc}} = (4.1 \pm 0.1)^{2} \text{ GeV}^{2}$$

$$s_{\Xi_{c}} = (3.2 \pm 0.1)^{2} \text{ GeV}^{2}$$

$$s_{\Xi_{c}} = (3.3 \pm 0.1)^{2} \text{ GeV}^{2}$$
The uncertainties are used to evaluate the numerical error.

$\Xi_{cc}^{++} \to \Xi_c^+ \pi^+$	Twist-2	Twist-3p	Twist- $3\sigma$	Total
$A_{\rm WE}$	$0.0084 \pm 0.0024$	$-0.077\pm0.01$	$-0.056 \pm 0.002$	$-0.124\pm0.011$
$B_{\rm WE}$	$-0.064 \pm 0.01$	$0.052\pm0.01$	$0.165 \pm 0.025$	$0.153 \pm 0.029$
$\Xi_{cc}^{++} \to \Xi_c^{+\prime} \pi^+$	Twist-2	Twist- $3p$	Twist- $3\sigma$	Total
$A'_{\rm WE}$	$0.0027 \pm 0.0005$	$0.0089\pm0.002$	$-0.018 \pm 0.0003$	$-0.0062 \pm 0.002$
$B'_{\rm WE}$	$0.0023 \pm 0.0006$	$0.052\pm0.016$	$0.011 \pm 0.003$	$0.066 \pm 0.016$

#### Numerical Result: W-exchange Amplitudes

$$\langle \Xi_{c}^{+(\prime)}(p-q)\pi^{+}(q)|\mathcal{H}_{\rm eff}(0)|\Xi_{cc}^{++}(p)\rangle_{\rm fac,nf} = i \ \bar{u}(p-q) \big[ A^{(\prime)\rm fac,nf} + B^{(\prime)\rm fac,nf}\gamma_{5} \big] u(p)$$

	$\Xi_{cc}^{++}\to\Xi_c^+\pi^+$	$A^{\mathrm{fac}}$	$A^{\mathrm{nf}}$	$A^{\mathrm{tot}}$	$B^{\mathrm{fac}}$	$B^{\mathrm{nf}}$	$B^{\mathrm{tot}}$
(	This work		$-16.67\pm1.41$			$20.47 \pm 3.89$	
	QCDSR	$-8.74\pm2.91$			$16.76\pm5.36$		
	LFQM + PM	7.40	-10.79	-3.38	15.06	-18.91	-3.85
	3LQM	-8.13	10.50	3.37	-12.97	18.53	5.56
	NRQM + PM	7.38	0	7.38	16.77	24.95	41.72
	HQET + PM	9.52	0	9.52	19.45	24.95	44.40
fac+nf ≺	$\Xi_{cc}^{++}\to\Xi_c^{+\prime}\pi^+$	$A'^{ m fac}$	$A'^{ m nf}$	$A'^{\rm tot}$	$B'^{ m fac}$	$B'^{ m nf}$	$B'^{ m tot}$
	This work		$-0.83\pm0.28$			$8.86 \pm 2.16$	
	QCDSR	$-3.55\pm0.68$			$34.13 \pm 11.6$		
	LFQM + PM	4.49	-0.04	4.45	48.50	-0.06	48.44
	3LQM	-4.34	-0.11	-4.45	-37.59	-1.37	-38.96
	NRQM + PM	4.29	0	4.29	53.65	0	53.65
	HQET + PM	5.10	0	5.10	62.37	0	62.37

in unit  $10^{-2}G_F$  GeV<sup>2</sup>

#### "fac" from literature and "nf" from this work

Method	$A^{ m tot}$	$B^{\mathrm{tot}}$	$\mathcal{B}(\Xi_{cc}^{++}\to\Xi_c^+\pi^+)$	$A'^{ m tot}$	$B'^{ m tot}$	$\mathcal{B}(\Xi_{cc}^{++}\to\Xi_c^{+\prime}\pi^+)$	$ \mathcal{B}'/\mathcal{B} $
QCDSR+LCSR	$-25.4\pm4.32$	$37.23 \pm 9.25$	$40\pm14~\%$	$-4.38\pm0.96$	$42.99 \pm 13.76$	$3.91{\pm}2.5~\%$	$0.098 \pm 0.14$
LFQM+LCSR	$-9.27 \pm 1.41$	$35.53 \pm 3.89$	$7.54 \pm 2.22~\%$	$3.66\pm0.28$	$57.36 \pm 2.16$	$5.83 \pm 0.5~\%$	$0.77\pm0.42$
3LCQM+LCSR	$-24.8\pm1.41$	$7.5\pm3.89$	$35.55 \pm 4.29~\%$	$-5.17\pm0.28$	$-28.73 \pm 2.16$	$2.75 \pm 0.35 \%$	$0.08\pm0.02$
NRQM+LCSR	$-9.29 \pm 1.41$	$37.24 \pm 3.89$	$7.82 \pm 2.25~\%$	$3.46\pm0.28$	$62.51 \pm 2.16$	$6.70 \pm 0.54~\%$	$0.85\pm0.44$
HQET+LCSR	$-7.18 \pm 1.41$	$39.92 \pm 3.89$	$6.22 \pm 1.94~\%$	$4.27\pm0.28$	$71.23 \pm 2.16$	$8.85 \pm 0.62~\%$	$1.42\pm0.78$
LFQM+PM	-3.83	3.85	0.69~%	4.45	48.44	4.65~%	6.74
3LCQM	3.37	5.56	0.71~%	-4.45	-38.96	3.39~%	4.77
HQET+PM	7.38	41.72	6.64~%	4.29	53.65	5.39~%	0.81
NRQM+PM	9.52	44.40	9.19%	5.1	62.37	7.34~%	0.8
$\mathrm{FSR}(\eta=1.0)$			7.11%			4.72~%	0.66
$\mathrm{FSR}(\eta=1.5)$			8.48%			4.72~%	0.56
$\mathrm{FSR}(\eta=2.0)$			10.75%			4.74~%	0.44

#### **Final state rescattering**

Chin. Phys. C 45, no.5, 053105 (2021)

## The interference between $B^{fac}$ and $B^{nf}$ tends to be constructive

#### **Consistent with experiment**

 $({\cal B}'/{\cal B})_{\rm expt} = 1.41 \pm 0.17 \pm 0.1$ 

- > The W-exchange amplitudes of  $\Xi_{cc}^{++} \rightarrow \Xi_{c}^{+(\prime)}\pi^{+}$  are calculated by light-cone sum rules.
- The possible branching fractions are obtained by combining our W-exchange amplitudes with the factorizable amplitudes from various theoretical works in the literature.
- > One of the possible branching fractions is consistent with the experimental result. The interference between  $B^{fac}$  and  $B^{nf}$  tends to be constructive.

## Thank you for your attention !