超子类时电磁形状因子及其振荡行为

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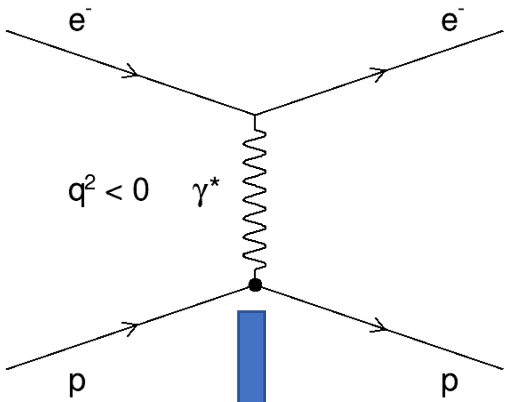
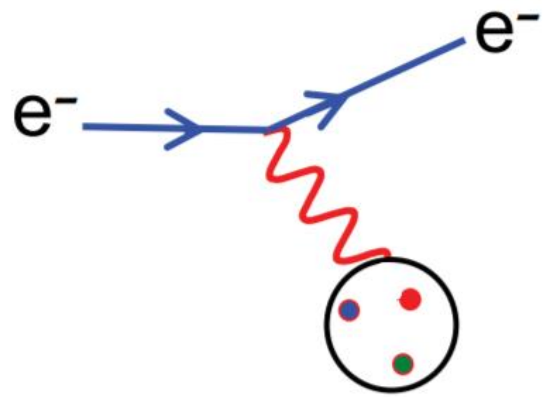
目录

电磁形状因子 (space-like)





$$\langle p_f | \hat{J}^{\mu}(0) | p_i \rangle = \overline{u}(p_f) \left[F_1(q^2)\gamma^{\mu} - F_2(q^2)\frac{\sigma^{\mu\nu} q_{\nu}}{2M} \right] u(p_i)$$

$$\Gamma^{\mu}(q^2) = \gamma^{\mu} F_1^{p}(q^2) + i\frac{F_2^{p}(q^2)}{2M_p}\sigma^{\mu\nu} q_{\nu}$$

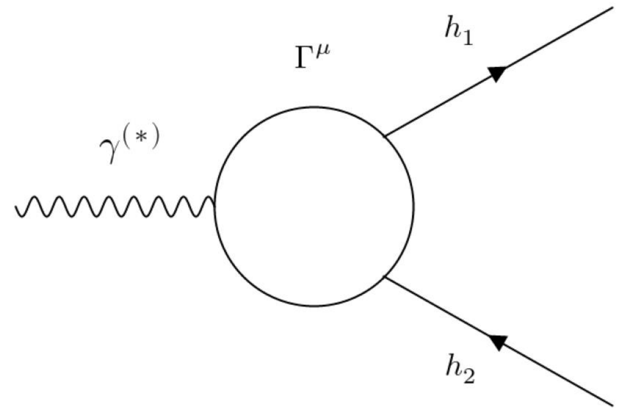
F_1^N和F_2^N是Dirac和Pauli形状因子

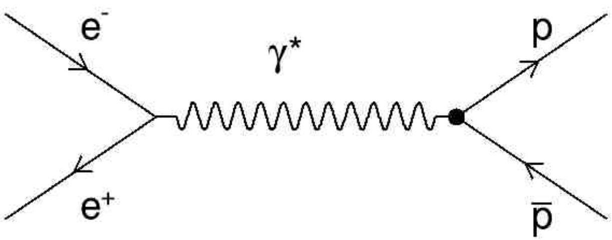
$$G_E^N(Q^2) = F_1^N(Q^2) - \tau F_2^N(Q^2)\ ,\quad G_M^N(Q^2) = F_1^N(Q^2) + F_2^N(Q^2)\ ,\quad \tau = \frac{Q^2}{4M_N^2}$$

$$F_1^p(0) = 1\ ,\ F_1^n(0) = 0\ ,\quad F_2^p(0) = \kappa_p\ ,\ F_2^n(0) = \kappa_n$$

S. Pacetti, R. Baldini Ferroli and E. Tomasi-Gustafsson, ``Proton electromagnetic form factors: Basic notions, present achievements and future perspectives," **Phys. Rept. 550-551, 1-103 (2015).**

电磁形状因子 (time-like)

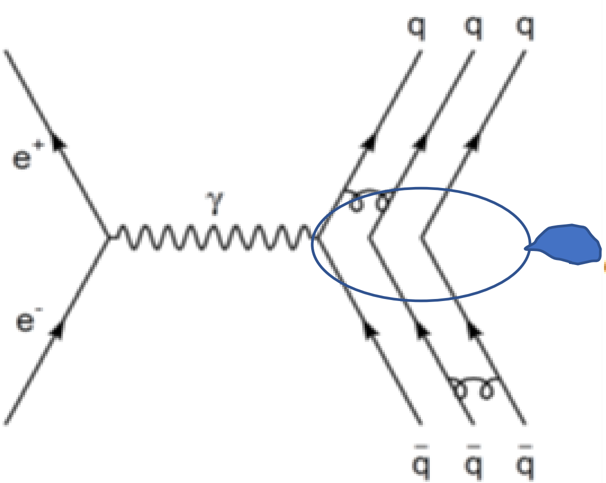




$$\left(\frac{d\sigma}{d\Omega}\right)^{th}_{e^+e^-\to N\bar{N}} = \frac{\alpha^2\beta}{4q^2} C_N(q^2)\left\{|G_M^N(q^2)|^2(1+\cos^2\theta)+|G_E^N(q^2)|^2\frac{1}{\tau}\sin^2\theta\right\}$$

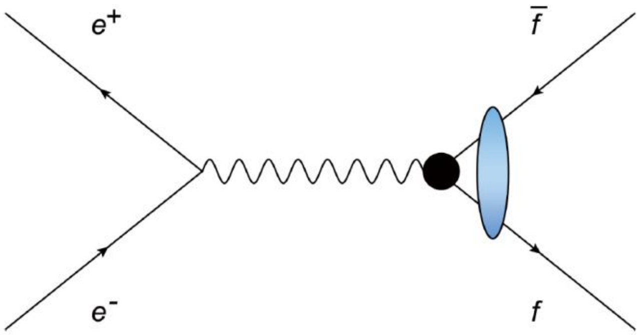
$$\sigma^{th}_{e^+e^-\to N\bar{N}} = \frac{\alpha^2\beta}{4q^2} C_N(q^2)\int d\Omega\left[|G_M^N(q^2)|^2(1+\cos^2\theta)+|G_E^N(q^2)|^2\frac{\sin^2\theta}{\tau}\right]$$

$$= \frac{4\pi\alpha^2\beta}{3q^2} C_N(q^2)\left[|G_M^N(q^2)|^2+\frac{|G_E^N(q^2)|^2}{2\tau}\right].$$



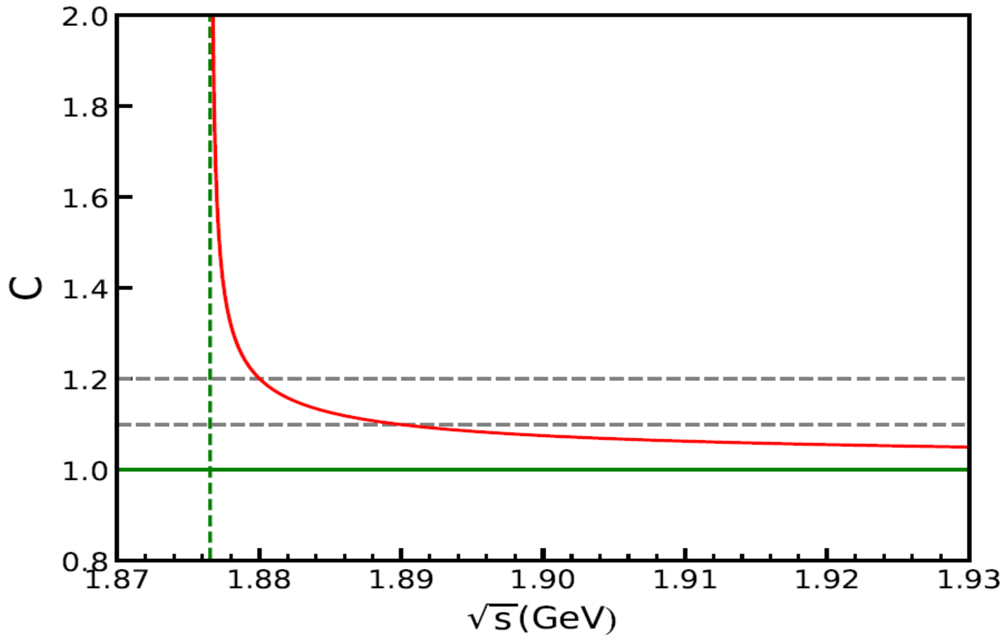
$$|G_{eff}(q^2)| = \sqrt{\frac{\sigma(q^2)}{\sigma_{point}(q^2)}} = \sqrt{\frac{|G_M(s)|^2+\frac{2M^2}{s}|G_E(s)|^2}{1+\frac{2M^2}{s}}}$$

索墨菲因子：末态库仑相互作用



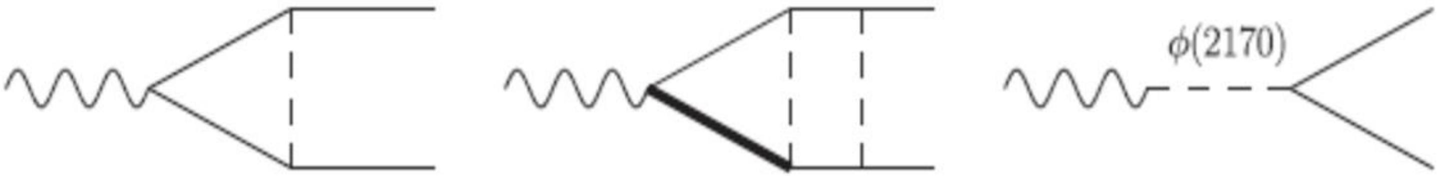
末态库仑相互作用示意图

$$\text{库仑因子 } C = \begin{cases} \dfrac{\pi\alpha}{\beta}\dfrac{1}{1-\exp(-\frac{\pi\alpha}{\beta})}, & \text{对于带电 } B\bar{B} \\ 1, & \text{对于中性 } B\bar{B} \end{cases}$$



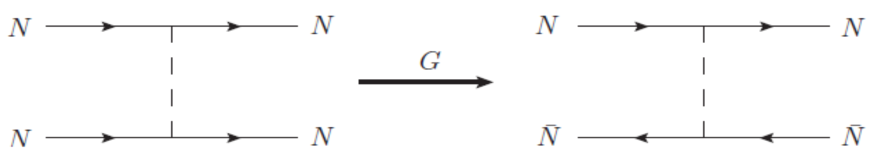
图：$p\bar{p}$产生过程的末态库仑修正因子 C 关于能量的依赖关系.

阈值增强：末态正反重子强相互作用



(a) (b) (c)

FSI-meson exchange *N*-excitation (Δ)* *First flavorless vector meson*

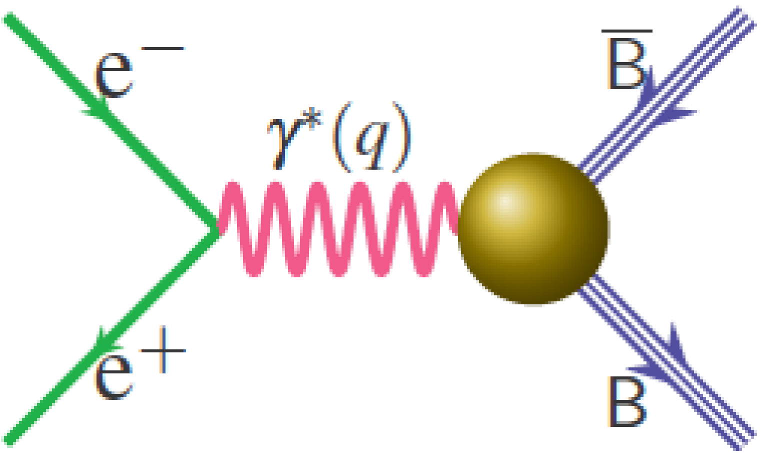


$$V_{\bar{N}N}^{\mathrm{OBE}} = (-1)^I V_{NN}^{\mathrm{OBE}}, \quad V_{\bar{N}N}^{\mathrm{TBE}} = (-1)^{I_1+I_2} V_{NN}^{\mathrm{TBE}}$$

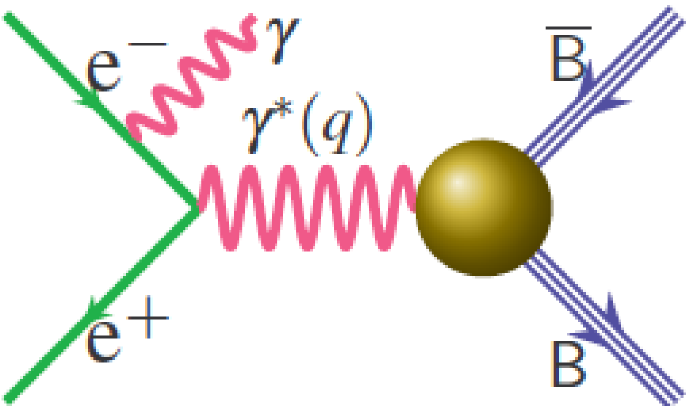
$$T_{L''L'}(p'',p';E_k) = V_{L''L'}(p'',p') + \sum_L \int \frac{dpp^2}{(2\pi)^3} V_{L''L}(p'',p) \frac{1}{2E_k - 2E_p + i0^+} T_{LL'}(p,p';E_k)$$

I.T. Lorenz, H.W. Hammer and U.G. Meissner, ``New structures in the proton-antiproton system,'' **Phys. Rev. D92, 034018 (2015).**

类时电磁形状因子的实验测量



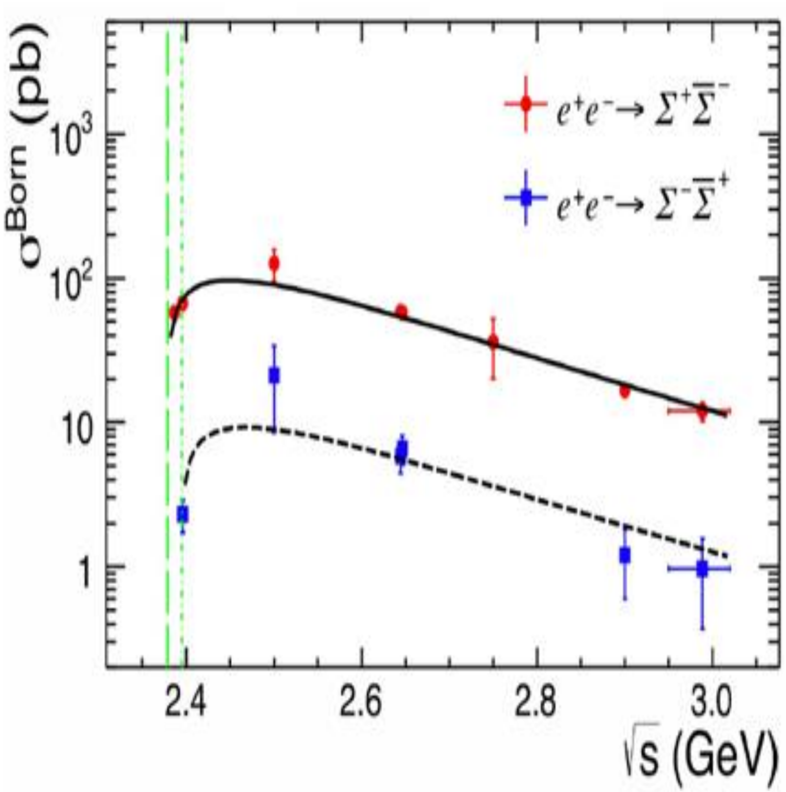
能量扫描

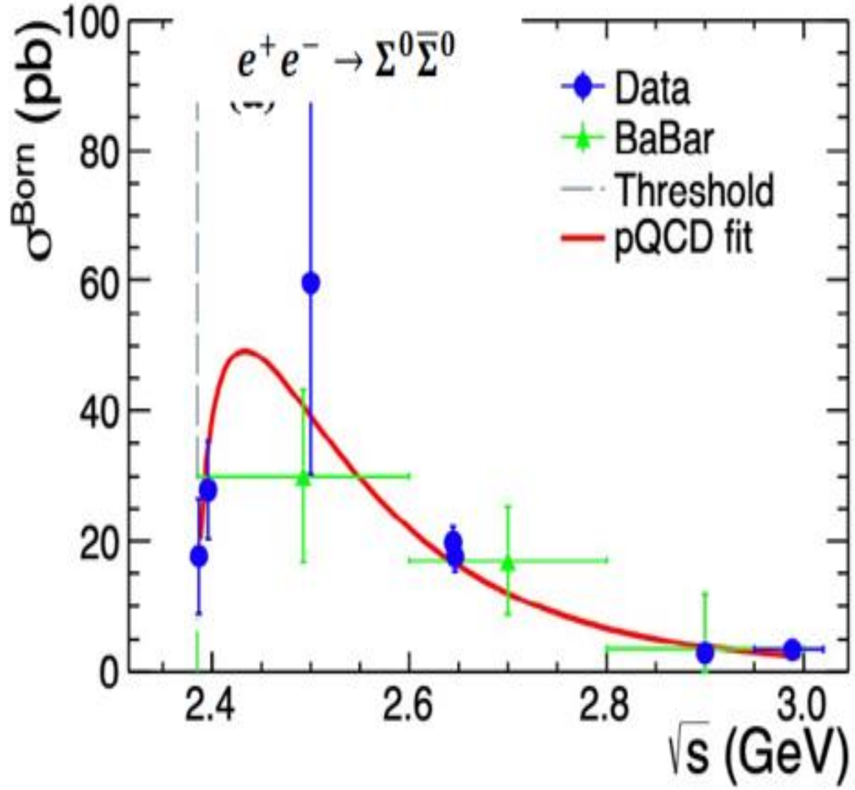


初态辐射

Σ超子的电磁形状因子---实验现状

$$\sigma(s)=\frac{4\pi\alpha^2\beta}{3s}C(s)\left[1+\frac{2M^2}{s}\right]|G_{eff}(s)|^2=\sigma_{point}(s)|G_{eff}(s)|^2.$$

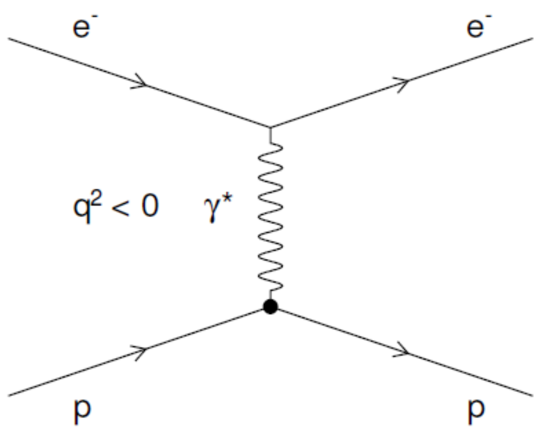


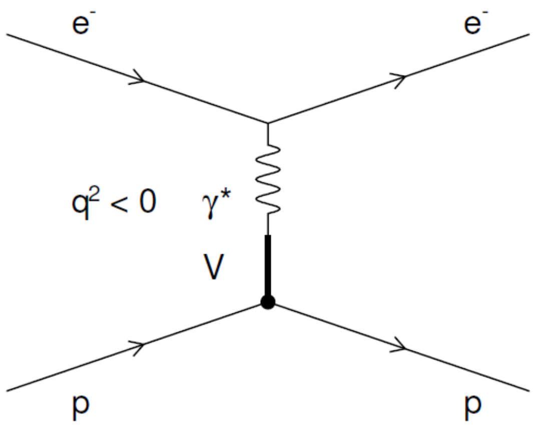


BESIII, **Phys. Lett. B 814, 136110 (2021)；Phys. Lett. B 831, 137187 (2022).**

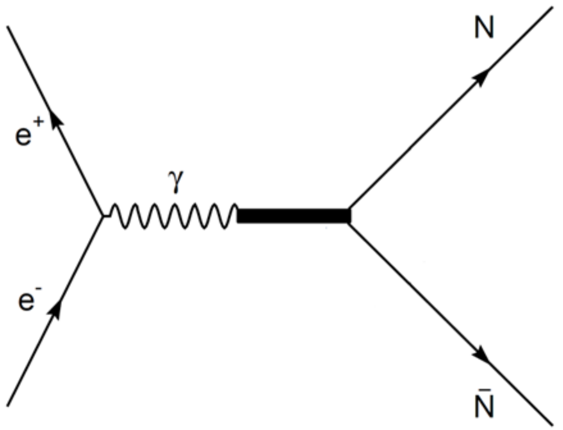
The ratio $\Sigma^+\bar{\Sigma}^-$: $\Sigma^0\bar{\Sigma}^0$: $\Sigma^-\bar{\Sigma}^+$ is about 9.7 ± 1.3 : 3.3 ± 0.7 : 1.

VMD:矢量介子为主模型





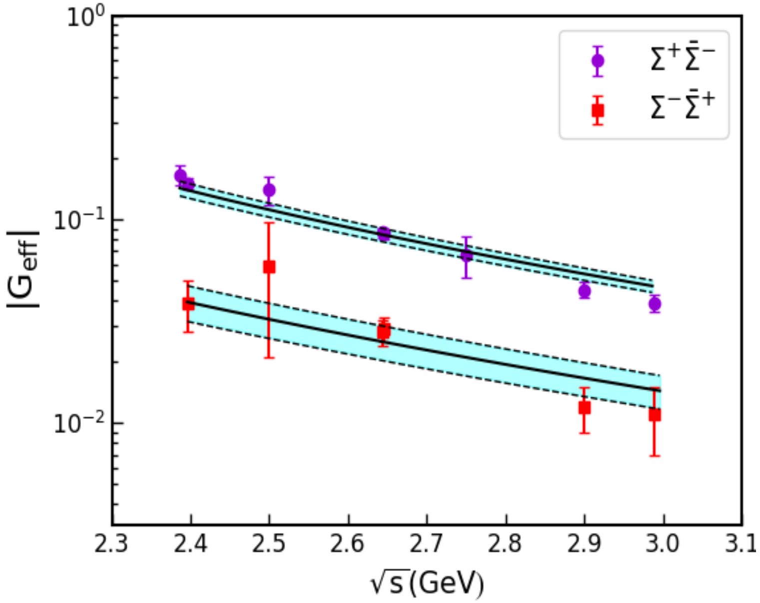
$$\mathcal{L}_{V\gamma} = \sum_V \frac{eM_V^2}{f_V} V_\mu A^\mu$$



$$L_{\mathrm{NNV}} = g\bar{\psi}\gamma_\mu\psi\varphi_V^\mu + \frac{\kappa}{4m}\bar{\psi}\sigma_{\mu\nu}\psi\left(\partial^\mu\varphi_V^\nu - \partial^\nu\varphi_V^\mu\right)$$

Σ^+和Σ^-的电磁形状因子(VMD)

$$
\begin{aligned}
F^S_{1\Sigma^+}\left(Q^2\right) &= \frac{1}{2}g_1\left(Q^2\right)\left[(1-\beta_\omega-\beta_\phi)+\beta_\omega\frac{m_\omega^2}{m_\omega^2+Q^2}+\beta_\phi\frac{m_\phi^2}{m_\phi^2+Q^2}\right] \\
F^V_{1\Sigma^+}\left(Q^2\right) &= \frac{1}{2}g_1\left(Q^2\right)\left[(1-\beta_\rho)+\beta_\rho\frac{m_\rho^2}{m_\rho^2+Q^2}\right] \\
F^S_{2\Sigma^+}\left(Q^2\right) &= \frac{1}{2}g_1\left(Q^2\right)\left[(4.224-\alpha_\phi-\alpha_\rho)\frac{m_\omega^2}{m_\omega^2+Q^2}+\alpha_\phi\frac{m_\phi^2}{m_\phi^2+Q^2}\right] \\
F^V_{2\Sigma^+}\left(Q^2\right) &= \frac{1}{2}g_1\left(Q^2\right)\left[\alpha_\rho\frac{m_\rho^2}{m_\rho^2+Q^2}\right] \\
F^S_{1\Sigma^-}\left(Q^2\right) &= \frac{1}{2}g_2\left(Q^2\right)\left[(-1-\beta_\omega-\beta_\phi)+\beta_\omega\frac{m_\omega^2}{m_\omega^2+Q^2}+\beta_\phi\frac{m_\phi^2}{m_\phi^2+Q^2}\right] \\
F^V_{1\Sigma^-}\left(Q^2\right) &= \frac{1}{2}g_2\left(Q^2\right)\left[(-1-\beta_\rho)+\beta_\rho\frac{m_\rho^2}{m_\rho^2+Q^2}\right] \\
F^S_{2\Sigma^-}\left(Q^2\right) &= \frac{1}{2}g_2\left(Q^2\right)\left[(-0.958-\alpha_\phi-\alpha_\rho)\frac{m_\omega^2}{m_\omega^2+Q^2}+\alpha_\phi\frac{m_\phi^2}{m_\phi^2+Q^2}\right] \\
F^V_{2\Sigma^-}\left(Q^2\right) &= \frac{1}{2}g_2\left(Q^2\right)\left[\alpha_\rho\frac{m_\rho^2}{m_\rho^2+Q^2}\right],
\end{aligned}
$$



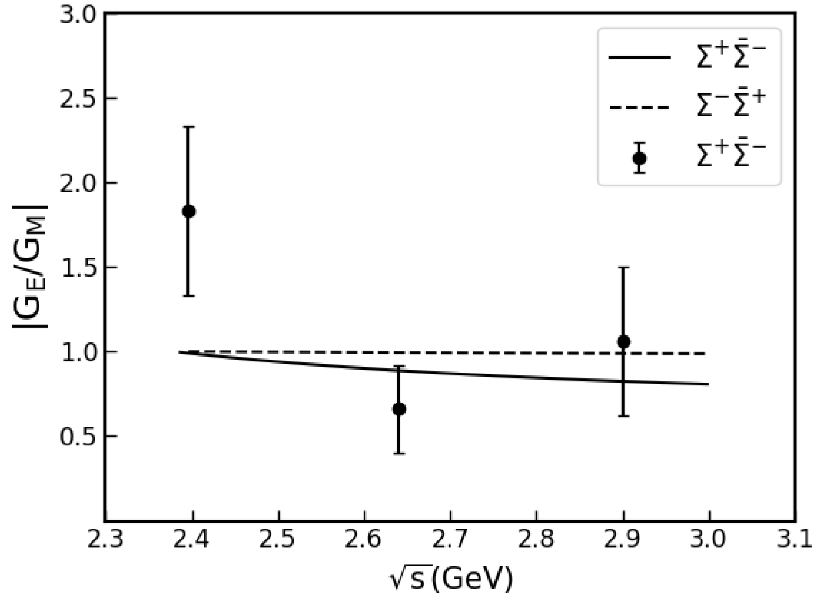


TABLE I: Parameters used in this work.

Parameter	Value	Parameter	Value
β_ρ	0.736	α_ρ	0.976
β_ϕ	-0.441	α_ϕ	1.035
β_ω	0.434		

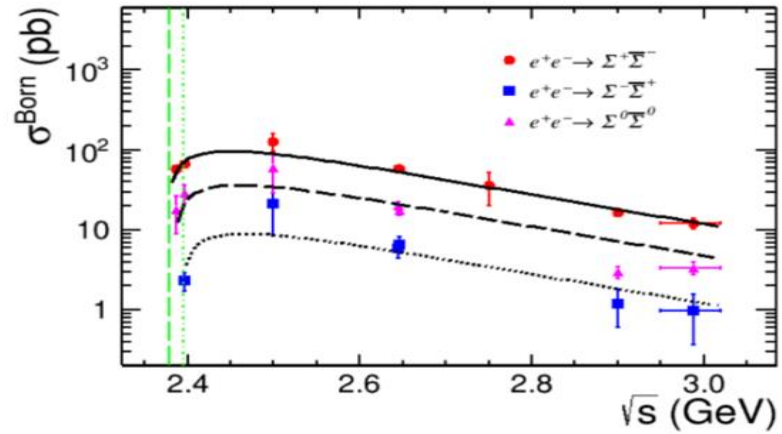
$$g_1(Q^2) = (1+\gamma_1 Q^2)^{-2},$$
$$g_2(Q^2) = (1+\gamma_2 Q^2)^{-2}.$$

$$G_E = F_1 + \tau F_2, \quad G_M = F_1 + F_2,$$

γ_1= 0.46$\pm$0.01 GeV^{-2} 和 γ_2= 1.18$\pm$0.13 GeV^{-2}。

Z. Y. Li and J. J. Xie, Commun. Theor. Phys. 73, 055201(2021).

同位旋分解？



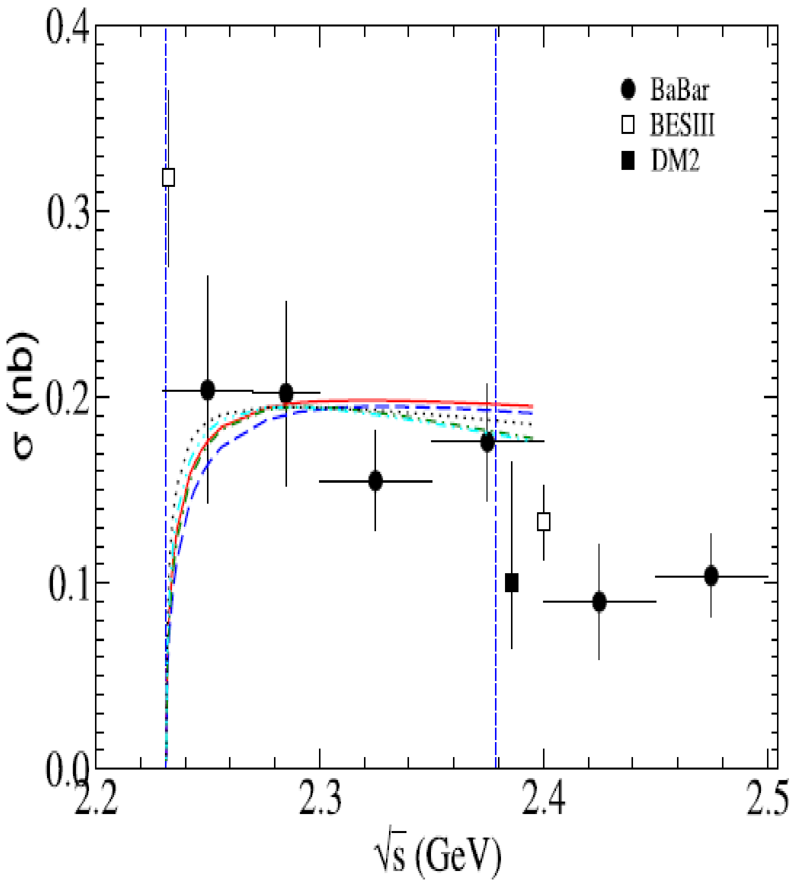
The ratio $\Sigma^+\bar{\Sigma}^- : \Sigma^0\bar{\Sigma}^0 : \Sigma^-\bar{\Sigma}^+$ is about $9.7 \pm 1.3 : 3.3 \pm 0.7 : 1$.

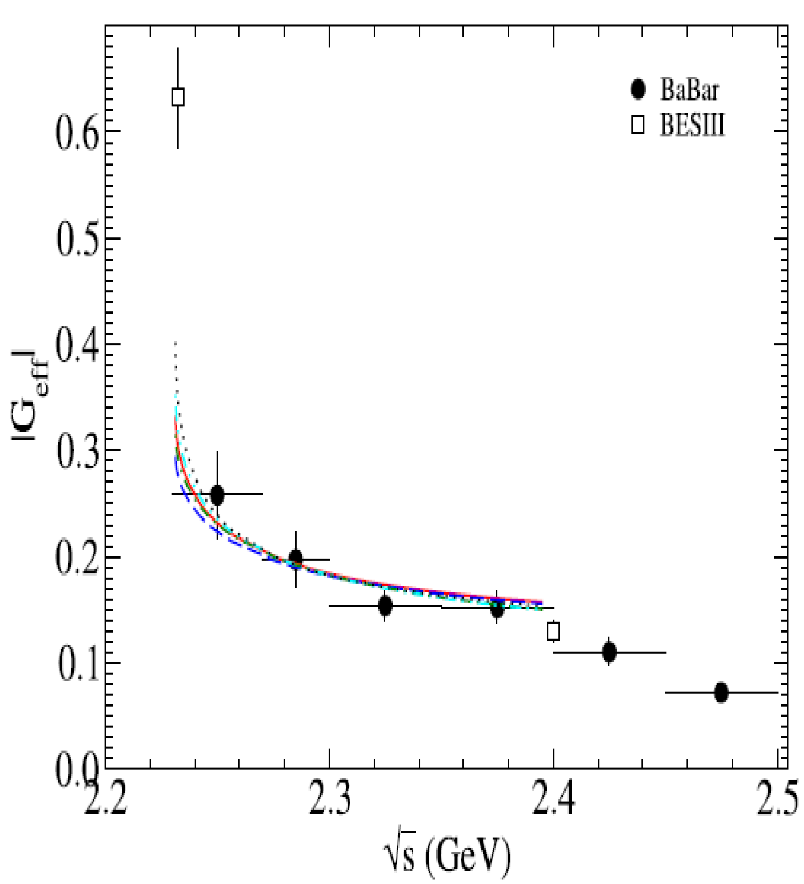
$$
\begin{aligned}
|1,0\rangle_A &= \frac{1}{\sqrt{2}}\{|1,1\rangle_\Sigma|1,-1\rangle_{\bar{\Sigma}} - |1,-1\rangle_\Sigma|1,1\rangle_{\bar{\Sigma}}\} \\
&= \frac{1}{\sqrt{2}}\{|\Sigma^+\bar{\Sigma}^-\rangle - |\Sigma^-\bar{\Sigma}^+\rangle\}
\end{aligned}
$$

$$
\begin{aligned}
|0,0\rangle &= \frac{1}{\sqrt{3}}\{|1,1\rangle_\Sigma|1,-1\rangle_{\bar{\Sigma}} - |1,0\rangle_\Sigma|1,0\rangle_{\bar{\Sigma}} + |1,-1\rangle_\Sigma|1,1\rangle_{\bar{\Sigma}}\} \\
&= \frac{1}{\sqrt{3}}\{|\Sigma^+\bar{\Sigma}^-\rangle - |\Sigma^0\bar{\Sigma}^0\rangle + |\Sigma^-\bar{\Sigma}^+\rangle\}
\end{aligned}
$$

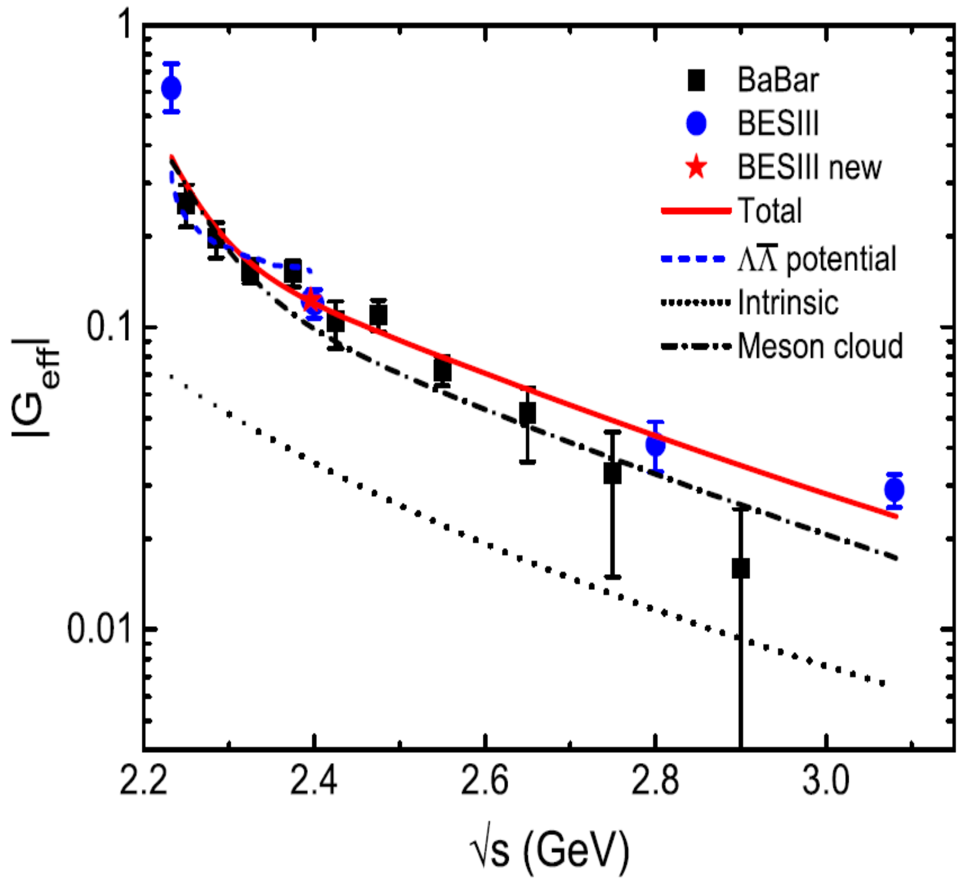
Very Preliminary

Λ的电磁形状因子(动机)





J. Haidenbauer and U. G. Meißner, Phys. Lett. B 761, 456-461(2016).



fit. In the present scenario, there are 16 experimental data and 10 free parameters. The value of intrinsic parameter γ is fitted to be 0.336 GeV^{-2} and the other parameters are summarized in Table II. It should be noticed that $g(q^2)$

State	Mass	Width	State	Mass	Width
$\omega(782)$ [55]	782	8.1	$\phi(1020)$ [56]	1019	4.2
$\omega(1420)$ [57]	1418	104	$\phi(1680)$ [57]	1674	165
$\omega(1650)$ [57]	1679	121	$\phi(2170)$ [58]	2171	128

Y. Yang, D. Y. Chen and Z. Lu, Phys. Rev. D 100, 073007 (2019).

Λ的电磁形状因子(新方案)

$$
\begin{aligned}
\mathrm{F}_1(\mathrm{Q}^2) &= \mathrm{g}(\mathrm{Q}^2)[-\beta_\omega - \beta_\phi + \beta_\omega \frac{\mathrm{m}_\omega^2}{\mathrm{m}_\omega^2 + \mathrm{Q}^2} + \beta_\phi \frac{\mathrm{m}_\phi^2}{\mathrm{m}_\phi^2 + \mathrm{Q}^2} + \beta_\mathrm{x} \frac{\mathrm{m}_\mathrm{x}^2}{\mathrm{m}_\mathrm{x}^2 + \mathrm{Q}^2}] \\
\mathrm{F}_2(\mathrm{Q}^2) &= \mathrm{g}(\mathrm{Q}^2)[(\mu_\Lambda - \alpha_\phi) \frac{\mathrm{m}_\omega^2}{\mathrm{m}_\omega^2 + \mathrm{Q}^2} + \alpha_\phi \frac{\mathrm{m}_\phi^2}{\mathrm{m}_\phi^2 + \mathrm{Q}^2} + \alpha_\mathrm{x} \frac{\mathrm{m}_\mathrm{x}^2}{\mathrm{m}_\mathrm{x}^2 + \mathrm{Q}^2}] \\
\mathrm{g}(\mathrm{Q}^2) &= 1/(1 + \gamma \mathrm{Q}^2)^2
\end{aligned}
$$

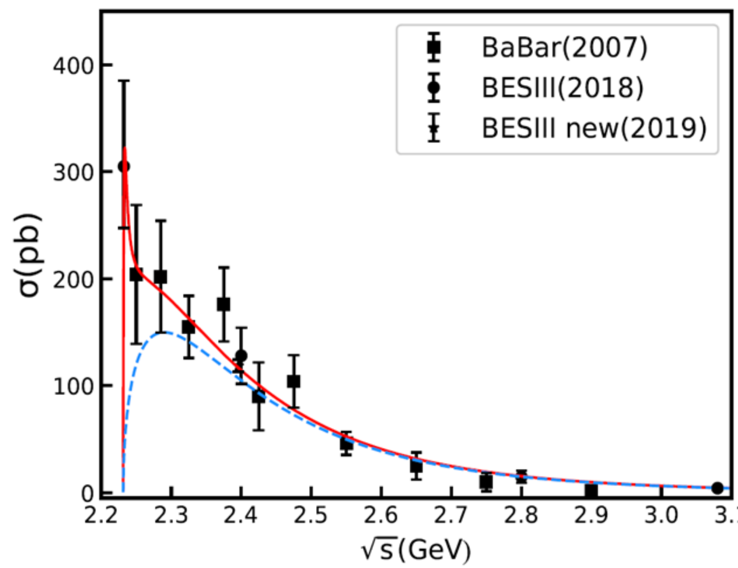
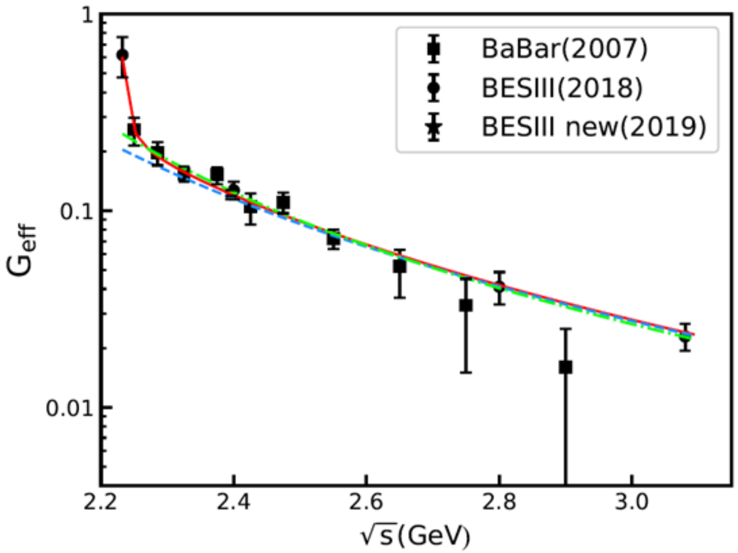


Figure: Cross section of the reaction $e^+e^- \to \bar{\Lambda}\Lambda$.

$$G_{\rm eff} = C_0 g(q^2) = \frac{C_0}{(1-\gamma q^2)^2}$$



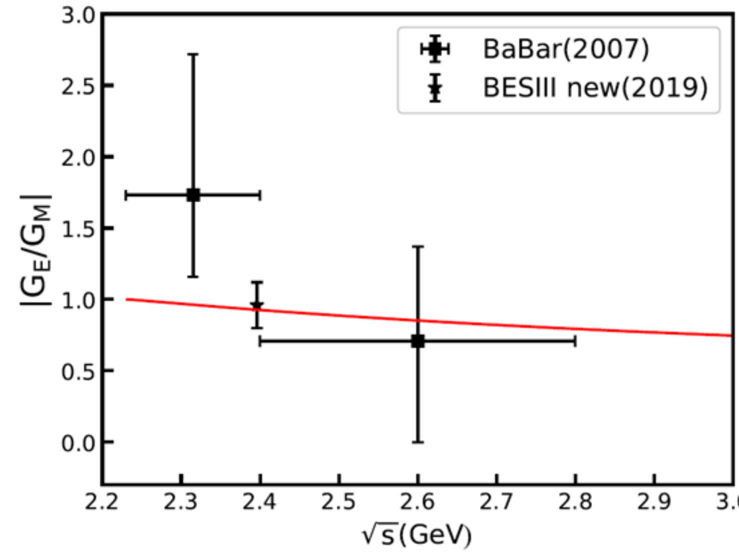
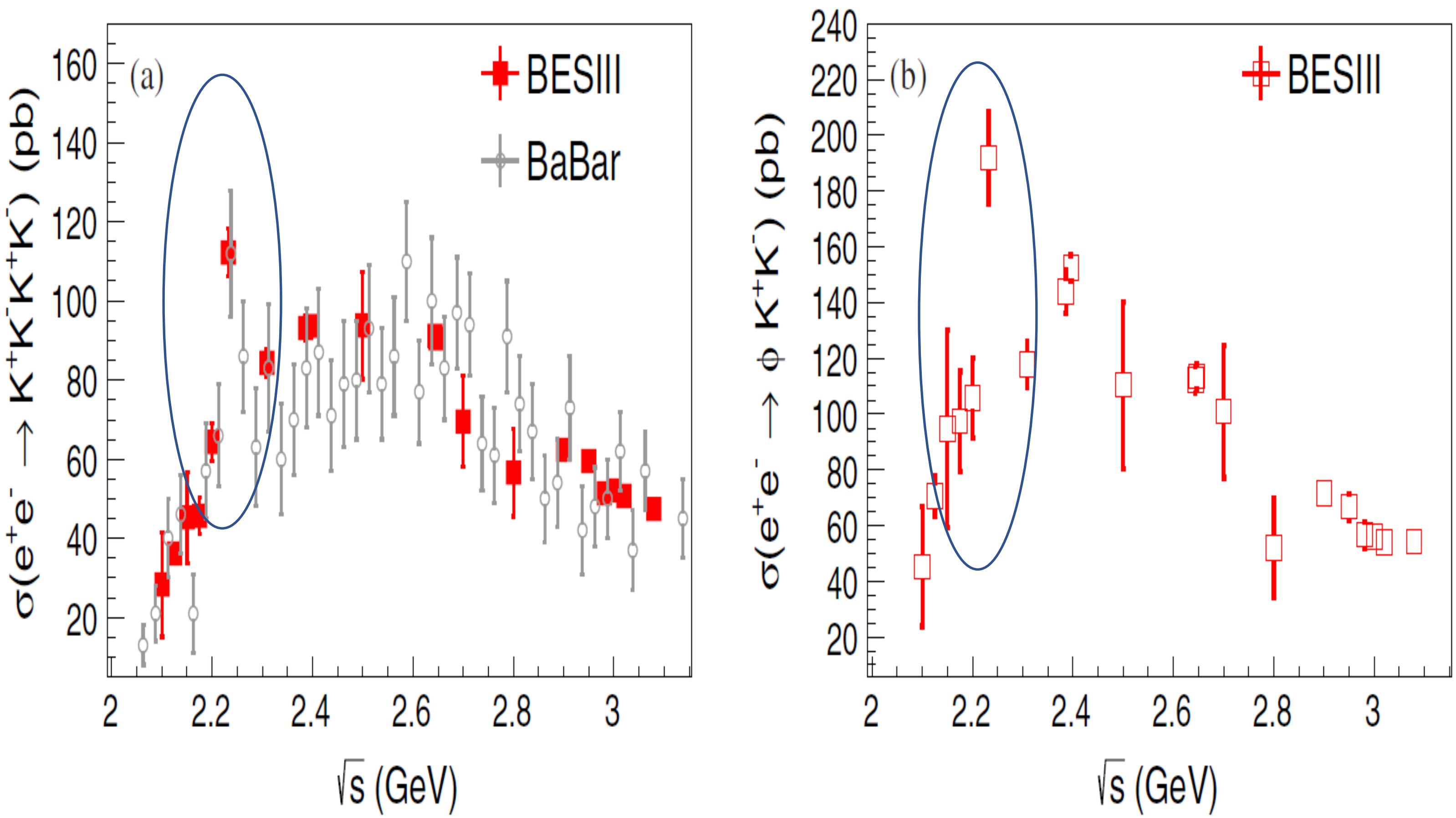


Table: Values of model parameters determined in this work.

Parameter	Value	Parameter	Value
γ (GeV^{-2})	0.43	β_ω	-1.13
β_ϕ	1.35	α_ϕ	-0.40
β_x	0.0015	m_x (MeV)	2230.9
Γ_x (MeV)	4.7		

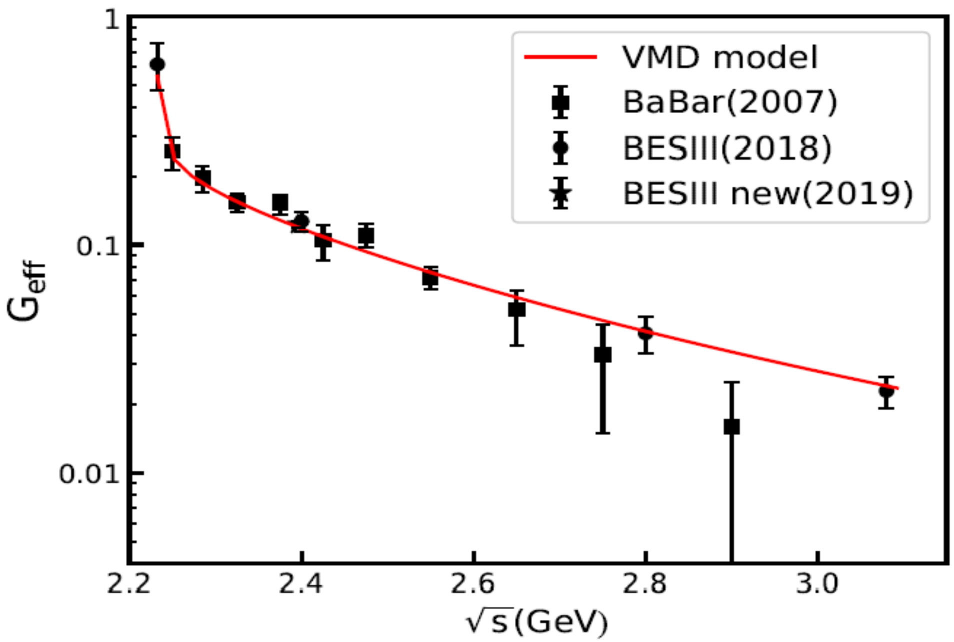
Z. Y. Li, A. X. Dai and J. J. Xie, Chin. Phys. Lett. 39, 011201 (2022).

X(2231)存在吗？在哪?



M. Ablikim, et al., Phys. Rev. D 100, 032009(2019).

Flatte function



Figure： Fitting result of $|G_{eff}|$ with Flatte.

$$\Gamma_x = \Gamma_0 + \Gamma_{\Lambda\bar{\Lambda}}\,(s) \qquad \Gamma_{\Lambda\bar{\Lambda}} = \frac{g^2}{4\pi}\sqrt{\frac{s}{4} - M_\Lambda^2}$$

Parameter	Value	Parameter	Value
γ (GeV^{-2})	0.57 ± 0.21	$\beta_{\omega\phi}$	-0.3 ± 0.31
β_x	-0.03 ± 0.09	m_x (MeV)	2237.7 ± 50.2
Γ_0 (MeV)	$8.8^{+75.9}_{-8.8}$	$g_{\Lambda\bar{\Lambda}}$	3.0 ± 1.9

$$\frac{d\sigma_i}{dm} = C\left|\frac{m_R\sqrt{\Gamma_o\Gamma_i}}{m_R^2 - m^2 - i m_R(\Gamma_{\pi\eta} + \Gamma_{K\bar{K}})}\right|^2$$

$$\Gamma_{\pi\eta} \simeq g_\eta q_\eta$$

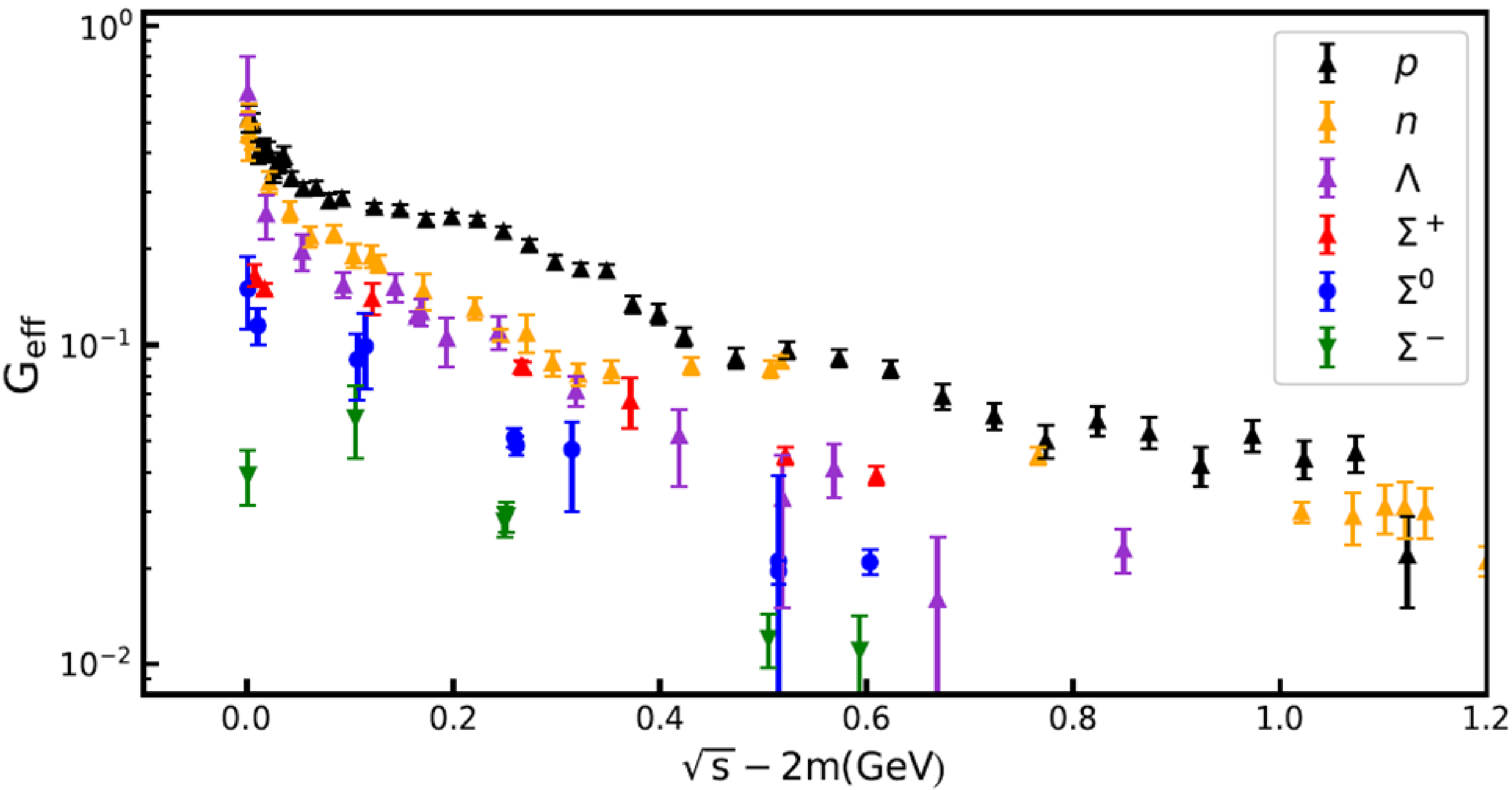
$$\Gamma_{K\bar{K}} = \begin{cases} g_K\sqrt{(1/4)m^2 - m_K^2} & \text{above threshold} \\ i g_K\sqrt{m_K^2 - (1/4)m^2} & \text{below threshold} \end{cases}$$

S.M. Flatte, Phys. Lett. B 63, 224-227 (1976).

On the other hand, if one takes a Flatté form for the total decay width of $\omega(1420)$, $\omega(1650)$, $\phi(1680)$, and $\phi(2170)$, the experimental data can also be well reproduced with a strong coupling of these resonances to the $\Lambda\bar{\Lambda}$ channel.

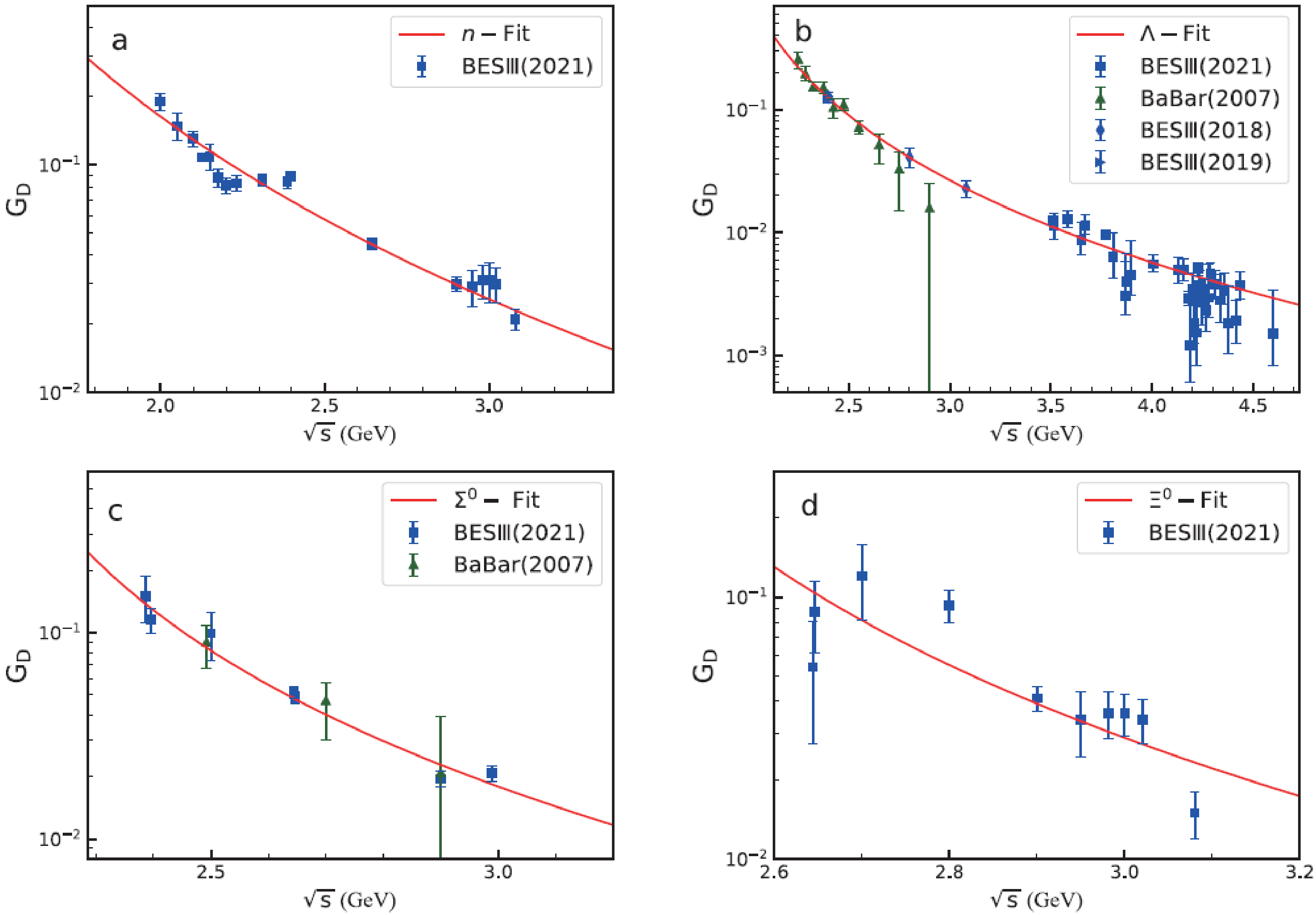
Z. Y. Li, A. X. Dai and J. J. Xie, Chin. Phys. Lett. 39, 011201 (2022).

有效形状因子的dipole衰减行为



图：p、n、Λ、Σ^+、Σ^0和Σ^-的类时有效形状因子测量结果。

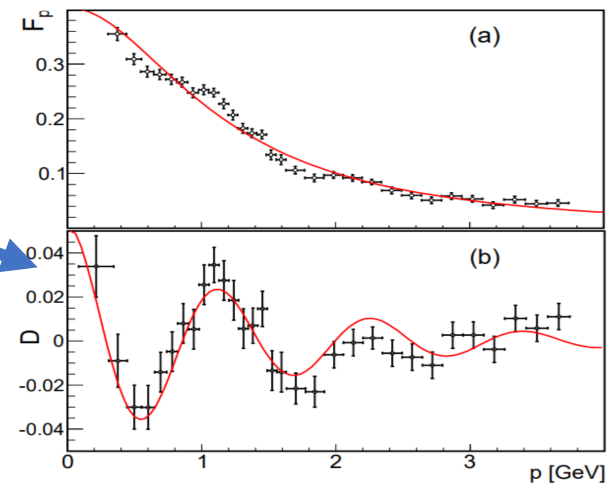
$$G_D(q^2) = \frac{c_0}{(1 - \gamma q^2)^2}$$



有效形状因子的振荡行为

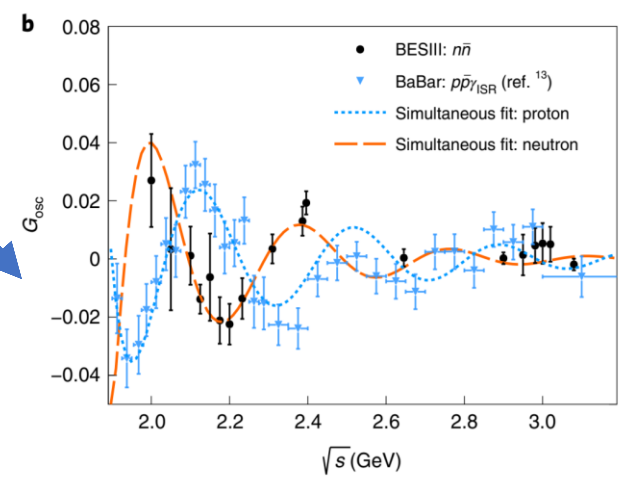
2015 年，Andrea Bianconi 等人在 [Phys. Rev. Lett., 2015, 114(23): 232301.] 中对质子类时的形状因子的实验数据进行了分析，提出质子的类时形状因子存在振荡的行为。

$$F_{3p}(s) = \frac{F_0}{\left(1+\frac{s}{m_a^2}\right)\left(1-\frac{s}{m_0^2}\right)^2},$$
$$F_{osc}(p(s)) = Ae^{-Bp}\cos(Cp+D).$$



2021年BESIII合作组以较高的精度测量了[Nature Phys., 2021, 17(11): 1200-1204]中子的形状因子，结果显示中子与质子的形状因子一样也存在振荡。

$$F_{osc}^{n,p} = A^{n,p}\exp\left(-B^{n,p}p\right)\cos\left(Cp+D^{n,p}\right)$$

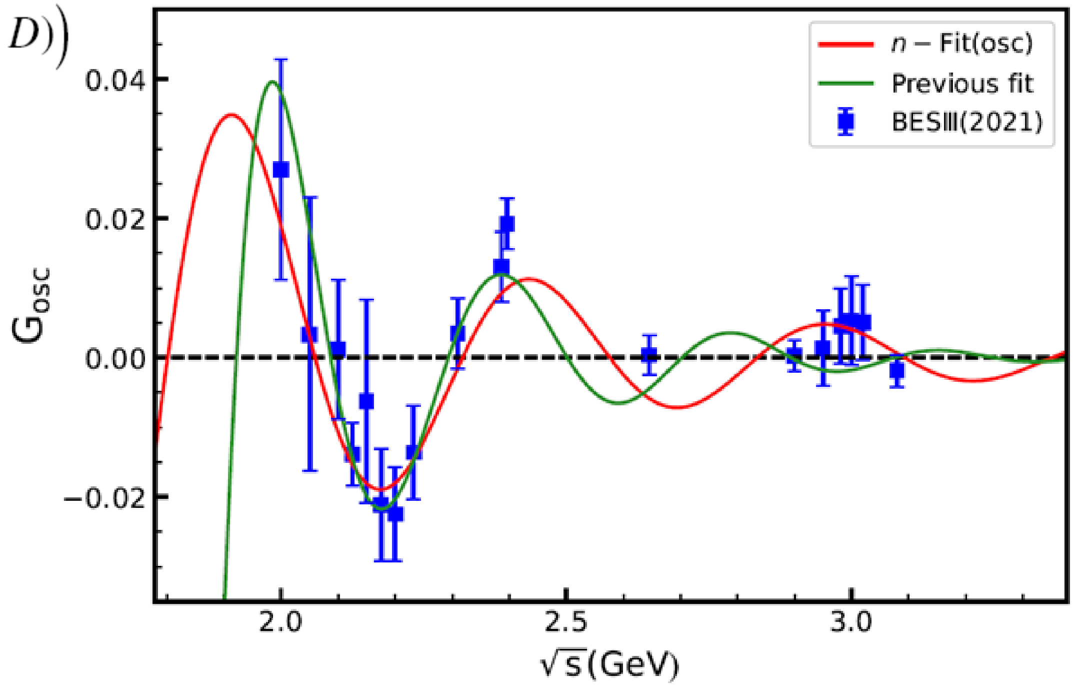


有效形状因子的振荡行为（新方案）

$$G_{osc} = A \cdot \frac{c_0}{(1-\gamma \cdot s)^2} \cdot \cos\left(C \cdot \sqrt{s} + D\right)$$

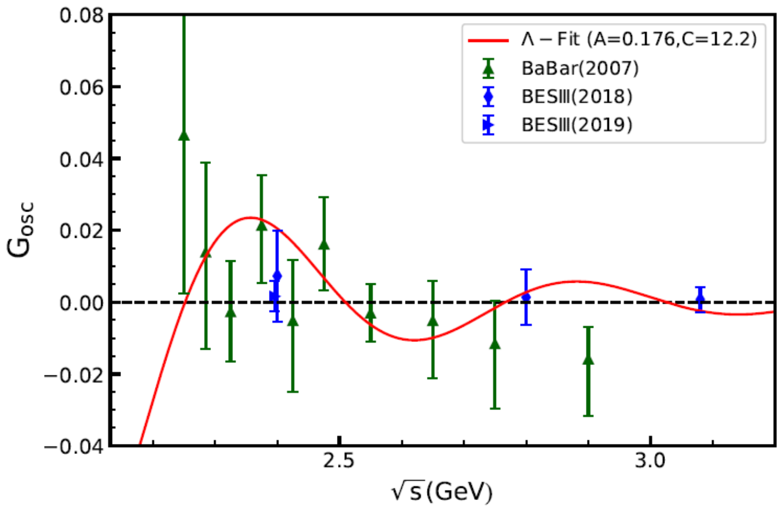
$$G_D(q^2) = \frac{c_0}{(1 - \gamma q^2)^2}$$

$$G_{\rm eff}(s) = G_{\rm D}(s) + G_{\rm osc}(s) = \frac{c_0}{(1-\gamma s)^2}\left(1 + A\cos(C\sqrt{s}+D)\right)$$

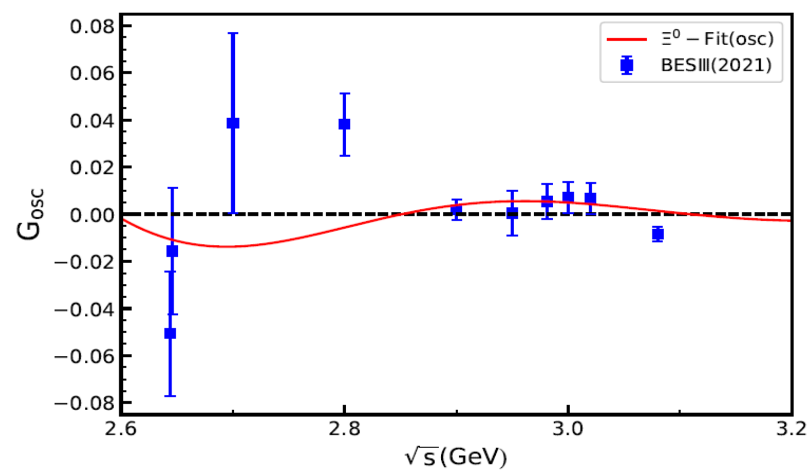


$$\boxed{data = G_{eff} = G_D + G_{osc}} \rightarrow \boxed{G_{osc} = data - G_D}$$

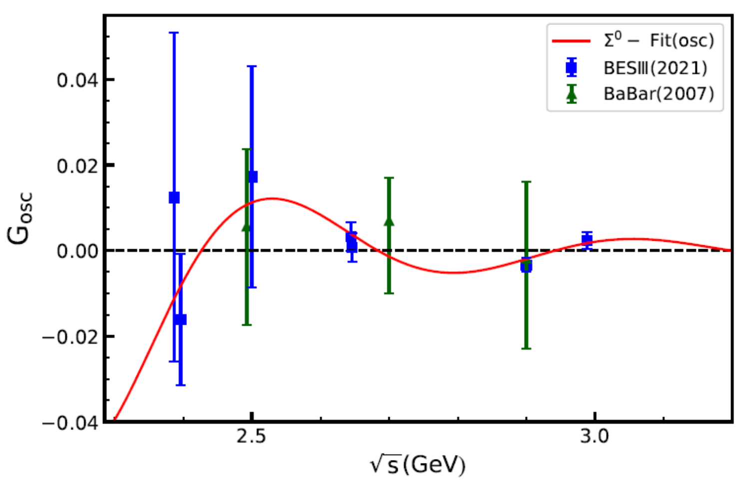
电中性重子有效形状因子的振荡行为



Λ超子有效形状因子的振荡部分拟合



Ξ^0超子有效形状因子的振荡部分拟合



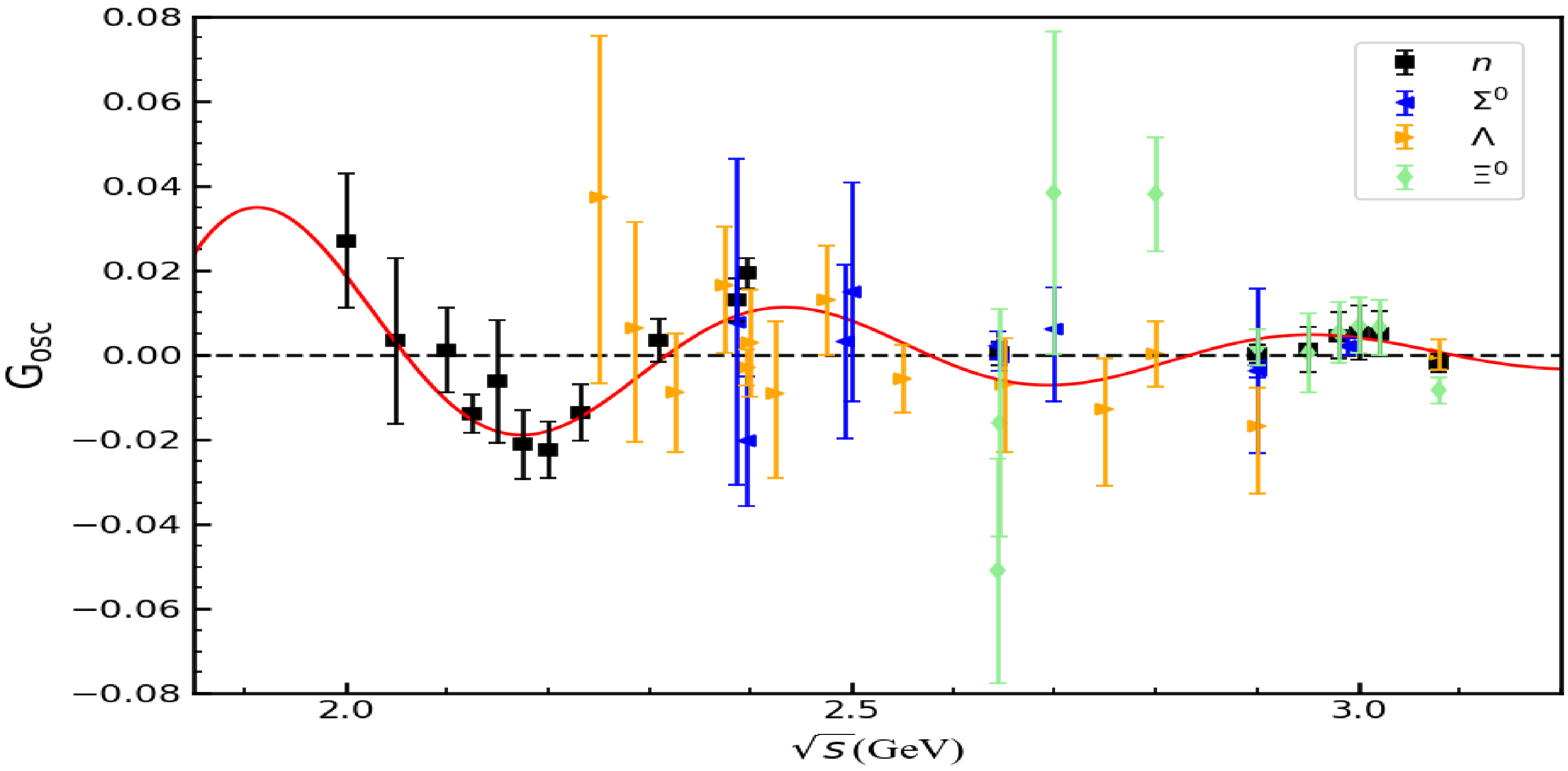
Σ^0超子有效形状因子的振荡部分拟合

1、加入振荡部分使得拟合的χ^2明显变小。

2、随着重子质量变大

参数A的值逐渐变大（振荡相对大小）

参数C的值逐渐变小（振荡角频率）



A.X. Dai, Z.Y. Li, L. Chang and J.J. Xie, Chin. Phys. C 46, 073104 (2022).

New experimental results

Eur. Phys. J. C (2022) 82:761
https://doi.org/10.1140/epjc/s10052-022-10696-0
THE EUROPEAN PHYSICAL JOURNAL C
Check for updates
Regular Article - Experimental Physics

Experimental study of the $e^+e^- \to n\bar{n}$ process at the VEPP-2000 e^+e^- collider with the SND detector

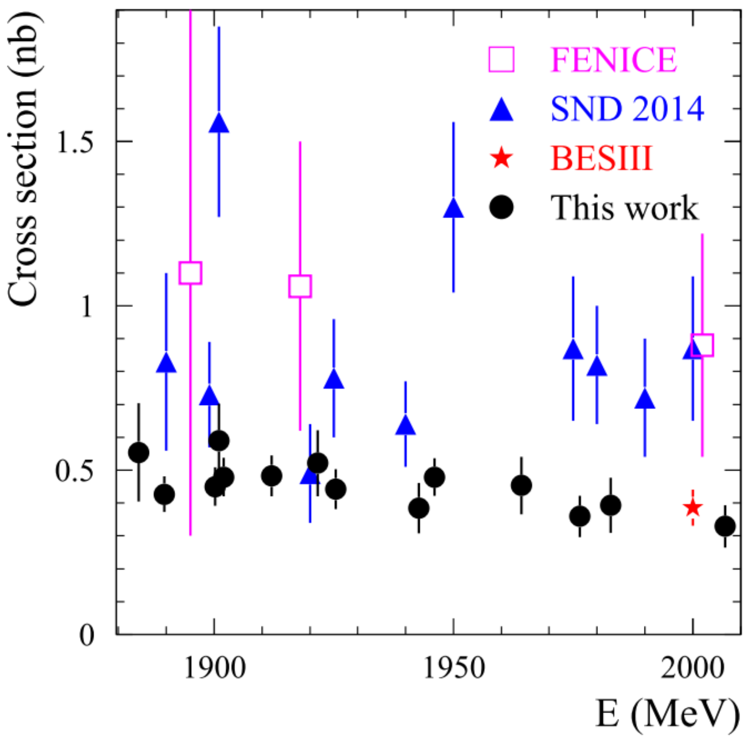
SND Collaboration

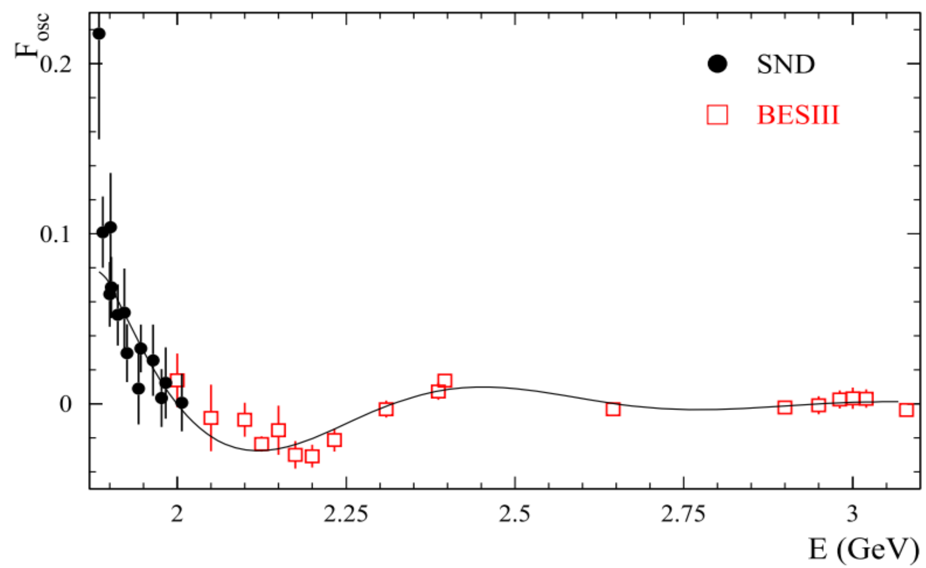
$$F(s) = F_0(s) + F_{osc}(s),$$

$$F_0(s) = \frac{\mathcal{A}_n}{\left[1 - s/0.71(\mathrm{GeV}^2)\right]^2},$$

$$F_{osc}(s) = A\exp(-Bp)\cos(Cp + D),$$

$$p = \sqrt{(s/2m_n - m_n)^2 - m_n^2}.$$





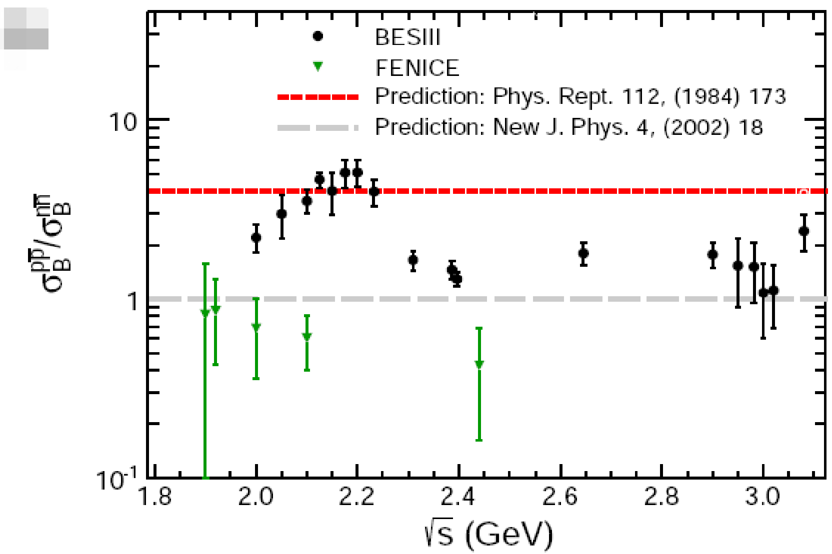
总结与展望

一、正反重子对产生截面阈值增强(平台)行为

末态相互作用　　　Flatte（强阈值耦合）

二、有效形状因子振荡行为

经验规律　　　机制尚不明确（矢量介子共振态？？）



一切都是刚刚起步！！！

We need more efforts, both on theoretical and experimental sides.

Thank you very much for your attention!