Searching for fully-heavy tetraquark states in QCD

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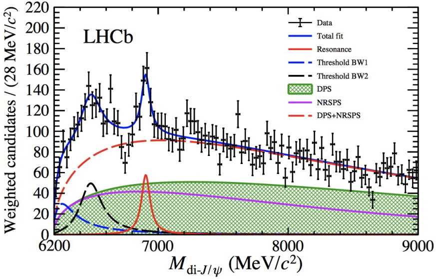
Outline

1. Observations of fully-charm tetraquark family
2. Moment sum rules for $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ tetraquarks
3. Decay properties of the $cc\bar{c}\bar{c}$ tetraquarks
4. Decay properties of the $bb\bar{b}\bar{b}$ tetraquarks
5. Summary

$X(6900)$: the first structure in $J/\psi J/\psi$ mass spectrum

LHCb observed some structures in the J/ψ-pair mass spectrum in 2020 (Sci.Bull.,2020,2020,65):



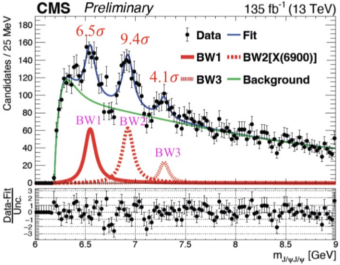
- The mass and width of $X(6900)$ are:(1) $M = 6905 \pm 11 \pm 7$ MeV, $\Gamma = 80 \pm 19 \pm 33$ MeV based on no-interference fit; (2) $M = 6886 \pm 11 \pm 11$ MeV, $\Gamma = 168 \pm 33 \pm 69$ MeV based on the simple model with interference.
- A broad structure next to threshold ranging from 6.2 to 6.8 GeV;
- Hint for another structure around 7.2 GeV with low significance.

CMS: $X(6600), X(6900), X(7300)$

CMS found 3 significant structures in the $J/\psi J/\psi$ mass spectrum (from **Yi Kai's slide**):

Final CMS model: 3 BWs + Background (null)

χ^2 Prob. = 1%
[6.2,7.8] GeV



Statistical significance based on:

2 ln(L_0/L_{max})

	BW1 (MeV)	BW2 (MeV)	BW3 (MeV)
m	6552 ± 10	6927± 9	7287± 19
Γ	124± 29	122± 22	95± 46
N	474± 113	492± 75	156± 56

- BW2[X(6900)] (>9.4σ) – confirmation
- Observation of BW1 (>5.7σ)
- Evidence for BW3 (>4.1σ)

Statistical significance only

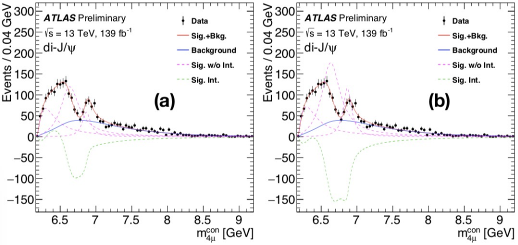
- **CMS found the family of fully-charm tetraquark states!**

ATLAS: $X(6200), X(6600), X(6900)$

ATLAS found 3 resonance structures in the $J/\psi J/\psi$ mass spectrum (from **Yue Xu's slide**):

Fit results in di-J/ψ channel

ATLAS-CONF-2022-040



- The **3rd peak** mass is consistent with the LHCb observed X(6900), with significance of **10σ**
- The broad structure at the lower mass could from other physical effects, e.g. feed-down from higher di-charmonium resonances

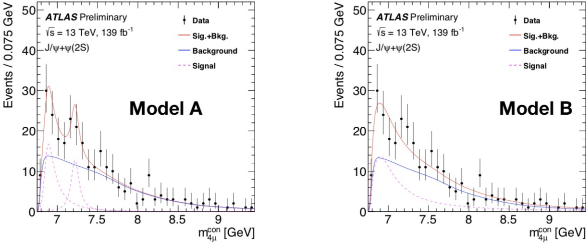
Extracted masses and widths (GeV)

(GeV)	m_0	Γ_0	m_1	Γ_1
di-J/ψ	$6.22 \pm 0.05^{+0.04}_{-0.05}$	$0.31 \pm 0.12^{+0.07}_{-0.08}$	$6.62 \pm 0.03^{+0.02}_{-0.01}$	$0.31 \pm 0.09^{+0.06}_{-0.11}$
	m_2	Γ_2	—	
	$6.87 \pm 0.03^{+0.06}_{-0.01}$	$0.12 \pm 0.04^{+0.03}_{-0.01}$	—	

ATLAS found 2 resonance structures in the $J/\psi\psi(2S)$ mass spectrum (from **Yue Xu's slide**):

Fit results in $J/\psi + \psi(2S)$ channel

ATLAS-CONF-2022-040



Extracted masses and widths (GeV)

(GeV)		m_3	Γ_3
J/ψ+ψ(2S)	model A	$7.22 \pm 0.03^{+0.02}_{-0.03}$	$0.10^{+0.13+0.06}_{-0.07-0.05}$
	model B	$6.78 \pm 0.36^{+0.35}_{-0.54}$	$0.39 \pm 0.11^{+0.11}_{-0.07}$

- In model A, the 1st peak could be related to X(6900) in the di-J/ψ channel. The significance of **2nd peak** (7.2 GeV) reaches 3.2σ, also hinted by LHCb in the di-J/ψ spectrum

Theoretical studies of four-heavy tetraquarks

Before Experiment:

- Quark-Gluon models: Prog. Theor. Phys. 54, 492 (1975); Zeit. Phys. C7, 317 (1981).
- Potential model: Phys.Rev. D25, 2370 (1982);32, 755 (1985); Phys. Lett. B123, 449 (1983).
- Hyperspherical harmonic formalism: Phys. Rev. D73, 054004 (2006).
- BS or Schrodinger Eqs: Phys.Rev.D86, 034004 (2012); Phys.Lett.B718, 545 (2012).
- Lattice QCD: Phys.Rev.D 97 (2018) 5, 054505;1709.09605
- QCD sum rules: PLB773 (2017) 247; EPJC77 (2017) 432
- Some others: PRD97 (2018) , 094015; EPJC77, 432 (2017);78 (2018) 8, 647; CPC43 (2019), 013105...

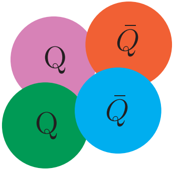
After Experiment: copious works to study their properties

- Diquark-antidiquark model: EPJC 80 (2020) 11, 1004;PRD103 (2021) 1, 014001; 104(2021) 114037; 014003; 014020;PLB811 (2020) 135952; 106 (2022) 096005; 106 (2022) 094019;Sci.Bull. 65 (2020) 1994-2000; PLB834 (2022) 137404;......
- Hadron-molecule model: PRD102 (2020) 9, 094001
- Tetracharm hybrid interpretation: PLB 817 (2021) 136339

Fully-heavy tetraquarks: $cc\bar{c}\bar{c}$

$cc\bar{c}\bar{c}$ Tetraquarks:

- They are far away from the mass range of the $c\bar{c}$ and $cq\bar{c}\bar{q}$ hadrons, can be clearly distinguished from the spectroscopy.
- The light mesons $(\pi, \rho, \omega, \sigma ...)$ can not be directly exchanged between two charmonia/bottomonia.
- The binding force comes from the short-range gluon exchange.
- A molecule configuration is not favored and thus the $cc\bar{c}\bar{c}$ is a good candidate for compact tetraquark.

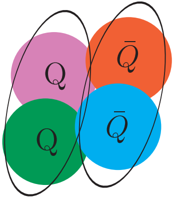


Tetraquark Sum Rules

- Study two-point correlation function of current $J(x)$ with the same quantum numbers with hadron state:

$$\Pi(q^2) = i\int d^4x e^{iq\cdot x}\langle\Omega|T[J(x)J^\dagger(0)]|\Omega\rangle$$

- Classify states $|X\rangle$ by coupling to current $\langle\Omega|J(x)|X\rangle \neq 0$
- Currents are probes of spectrum and might not overlap with state



Pauli principle operates for diquark structure:

Operators with $J^{PC} = 0^{-+}$ and 0^{--}:

$$J_1^\pm = Q_a^T C Q_b \bar{Q}_a \gamma_5 C \bar{Q}_b^T \pm Q_a^T C\gamma_5 Q_b \bar{Q}_a C \bar{Q}_b^T \,,$$
$$J_2^+ = Q_a^T C\sigma_{\mu\nu} Q_b \bar{Q}_a \sigma^{\mu\nu}\gamma_5 C \bar{Q}_b^T \,,$$

- Hadron level: described by the dispersion relation

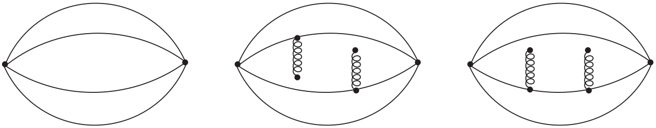
$$\Pi(q^2) = \frac{(q^2)^N}{\pi}\int \frac{\mathrm{Im}\Pi(s)}{s^N(s-q^2-i\epsilon)}ds + \sum_{n=0}^{N-1} b_n(q^2)^n,$$

$$\rho(s) = \frac{1}{\pi}\mathrm{Im}\Pi(s) = \sum_n \delta(s-m_n^2)\langle 0|J|n\rangle\langle n|J^\dagger|0\rangle$$

$$= f_X^2\delta(s-m_X^2) + \text{continuum},$$

- Quark-gluon level: evaluated via operator product expansion(OPE)

$$\Pi(s) = \Pi^{pert}(s) + \Pi^{\langle GG\rangle}(s) + ...,$$



- Define moments in Euclidean region $Q^2 = -q^2 > 0$:

$$M_n(Q_0^2) = \frac{1}{n!}\left(-\frac{d}{dQ^2}\right)^n \Pi(Q^2)|_{Q^2=Q_0^2}$$
$$= \int_{m_H^2}^{\infty} \frac{\rho(s)}{(s+Q_0^2)^{n+1}} ds = \frac{f_X^2}{(m_X^2+Q_0^2)^{n+1}} \left[1+\delta_n(Q_0^2)\right],$$

where $\delta_n(Q_0^2)$ contains the higher states and continuum.

- Ratio of the moments

$$r(n, Q_0^2) \equiv \frac{M_n(Q_0^2)}{M_{n+1}(Q_0^2)} = \left(m_X^2 + Q_0^2\right) \frac{1+\delta_n(Q_0^2)}{1+\delta_{n+1}(Q_0^2)}.$$

- Predict hadron mass

$$m_X = \sqrt{r(n, Q_0^2) - Q_0^2}$$

for sufficiently large n when $\delta_n(Q_0^2) \cong \delta_{n+1}(Q_0^2)$ for convergence.

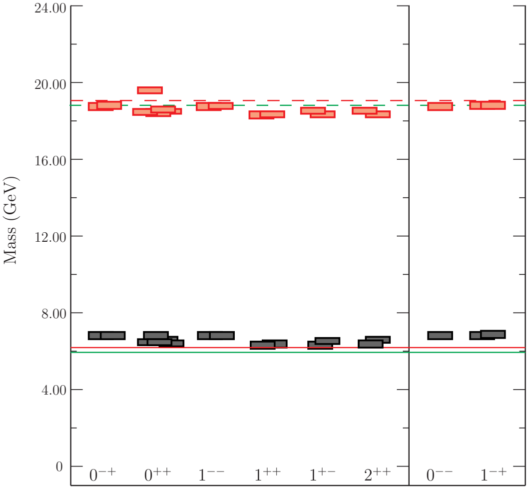
Good predictions for experimental observations:

PLB773(2017), 247-251

J^{PC}	Currents	m_{X_c}(GeV)	m_{X_b}(GeV)
0^{++}	J_1	6.44 ± 0.15	18.45 ± 0.15
	J_2	6.59 ± 0.17	18.59 ± 0.17
	J_3	6.47 ± 0.16	18.49 ± 0.16
	J_4	6.46 ± 0.16	18.46 ± 0.14
	J_5	6.82 ± 0.18	19.64 ± 0.14
1^{++}	$J_{1\mu}^{+}$	6.40 ± 0.19	18.33 ± 0.17
	$J_{2\mu}^{+}$	6.34 ± 0.19	18.32 ± 0.18
1^{+-}	$J_{1\mu}^{-}$	6.37 ± 0.18	18.32 ± 0.17
	$J_{2\mu}^{+}$	6.51 ± 0.15	18.54 ± 0.15
2^{++}	$J_{1\mu\nu}$	6.51 ± 0.15	18.53 ± 0.15
	$J_{2\mu\nu}$	6.37 ± 0.19	18.32 ± 0.17
0^{-+}	J_1^{+}	6.84 ± 0.18	18.77 ± 0.18
	J_2^{+}	6.85 ± 0.18	18.79 ± 0.18
0^{--}	J_1^{-}	6.84 ± 0.18	18.77 ± 0.18
1^{-+}	$J_{1\mu}^{+}$	6.84 ± 0.18	18.80 ± 0.18
	$J_{2\mu}^{+}$	6.88 ± 0.18	18.83 ± 0.18
1^{--}	$J_{1\mu}^{-}$	6.84 ± 0.18	18.77 ± 0.18
	$J_{2\mu}^{-}$	6.83 ± 0.18	18.77 ± 0.16

Our predictions in 2017 are consistent very good with the observations of LHCb, CMS and ATLAS:

- **The masses of $cc\bar{c}\bar{c}$ tetraquarks with $J^{PC} = 0^{++}, 2^{++}$ are agree with the mass of X(6200) and X(6600);**
- **The masses of $cc\bar{c}\bar{c}$ tetraquarks with $J^{PC} = 0^{-+}, 1^{-+}$are consistent with the mass of X(6900).**



Mass (GeV)
24.00
20.00
16.00
12.00
8.00
4.00
0
0^{-+}
0^{++}
1^{--}
1^{++}
1^{+-}
2^{++}
0^{--}
1^{-+}

$c\bar{c}c\bar{c}$ tetraquarks in $\mathbf{8}_{[c\bar{c}]} \otimes \mathbf{8}_{[c\bar{c}]}$:

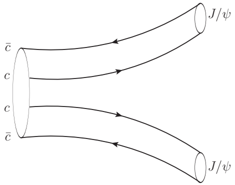
Phys.Rev.D 104 (2021) 11, 114037

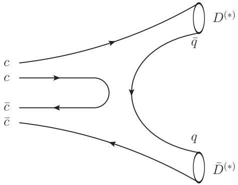
Current	J^{PC}	Mass(GeV)	
J_1	0^{++}	$6.51^{+0.18}_{-0.17}$	$18.15^{+0.14}_{-0.10}$
J_2	0^{++}	$6.34^{+0.15}_{-0.16}$	$18.13^{+0.13}_{-0.09}$
$J_{1\mu}$	1^{+-}	$6.44^{+0.16}_{-0.17}$	$18.14^{+0.14}_{-0.09}$
$J_{\mu\nu}$	2^{++}	$6.44^{+0.17}_{-0.17}$	$18.15^{+0.14}_{-0.09}$
η_1	0^{-+}	$7.00^{+0.23}_{-0.20}$	$18.45^{+0.15}_{-0.11}$
η_2	0^{-+}	$7.02^{+0.24}_{-0.20}$	$18.54^{+0.16}_{-0.12}$
η_3	0^{--}	$7.00^{+0.23}_{-0.20}$	$18.47^{+0.15}_{-0.11}$
$\eta_{1\mu}$	1^{--}	$6.98^{+0.23}_{-0.20}$	$18.46^{+0.15}_{-0.11}$
$\eta_{2\mu}$	1^{--}	$7.10^{+0.24}_{-0.21}$	$18.46^{+0.15}_{-0.11}$
$\eta_{3\mu}$	1^{-+}	$7.00^{+0.23}_{-0.20}$	$18.56^{+0.16}_{-0.11}$
$\eta_{4\mu}$	1^{-+}	$7.14^{+0.25}_{-0.21}$	$18.79^{+0.18}_{-0.13}$

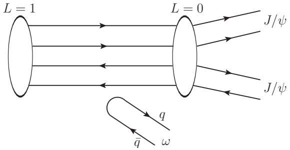
- Almost the same mass spectra with diquark-antidiquark configurations!
- A $c\bar{c}c\bar{c}$ tetraquark is predicted close to $X(7200)$.

Decay behavior: $cc\bar{c}\bar{c}$ tetraquarks

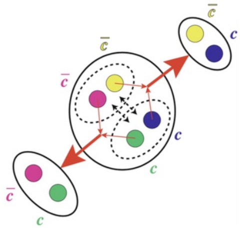
- $cc\bar{c}\bar{c} \to (c\bar{c}) + (c\bar{c})$: charm quark pair rearrangement. **Allowed.**
- $cc\bar{c}\bar{c} \to (ccq) + (\bar{c}\bar{c}\bar{q})$: kinematically forbidden.
- $cc\bar{c}\bar{c} \to (cqq) + (\bar{c}\bar{q}\bar{q})$: suppressed by two light quark pair creation.
- $cc\bar{c}\bar{c} \to (q\bar{c}) + (c\bar{q})$: possible via a heavy quark pair annihilation and a light quark pair creation, with large phase space.
- $cc\bar{c}\bar{c}(L=1) \to cc\bar{c}\bar{c}(L=0) + (q\bar{q})_{I=0}$: OZI forbidden.







Strong decays into di-charmonia



PLB773(2017), 247-251;
Sci.Bull.65,2020, 1994-2000

J^{PC}	S-wave	P-wave
0^{++}	$\eta_c(1S)\eta_c(1S)$, $J/\psi J/\psi$	$\eta_c(1S)\chi_{c1}(1P)$, $J/\psi h_c(1P)$
0^{-+}	$\eta_c(1S)\chi_{c0}(1P)$, $J/\psi h_c(1P)$	$J/\psi J/\psi$
0^{--}	$J/\psi\chi_{c1}(1P)$	$J/\psi\eta_c(1S)$
1^{++}	–	$J/\psi h_c(1P)$, $\eta_c(1S)\chi_{c1}(1P)$, $\eta_c(1S)\chi_{c0}(1P)$
1^{+-}	$J/\psi\eta_c(1S)$	$J/\psi\chi_{c0}(1P)$, $J/\psi\chi_{c1}(1P)$, $\eta_c(1S)h_c(1P)$
1^{-+}	$J/\psi h_c(1P)$, $\eta_c(1S)\chi_{c1}(1P)$	$J/\psi J/\psi$
1^{--}	$J/\psi\chi_{c0}(1P)$, $J/\psi\chi_{c1}(1P)$, $\eta_c(1S)h_c(1P)$	$J/\psi\eta_c(1S)$

Strong decays into di-charmonia

Relative branching ratios by Fierz rearrangement: Sci.Bull.65(2020)1994

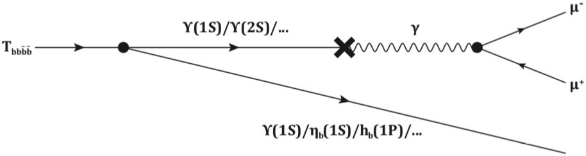
J^{PC}	Current	Decay channels												
		$J/\psi J/\psi$	$J/\psi\psi'$	$\eta_c\eta_c$	$\eta_c\eta_c'$	$J/\psi h_c$	$\eta_c\chi_{c0}$	$\eta_c\chi_{c1}$	$J/\psi\eta_c$	$J/\psi\eta_c'$	$\psi'\eta_c$	$J/\psi\chi_{c0}$	$J/\psi\chi_{c1}$	$\eta_c h_c$
0^{++}	$J_1^{0^{++}}$	1	⋯	0.45	⋯	⋯	⋯	2×10^{-5}	⋯	⋯	⋯	⋯	⋯	⋯
	$J_2^{0^{++}}$	1	⋯	4.1	⋯	⋯	⋯	9×10^{-5}	⋯	⋯	⋯	⋯	⋯	⋯
1^{+-}	$J_{3\alpha}^{1^{+-}}$	⋯	⋯	⋯	⋯	⋯	⋯	⋯	1	⋯	⋯	⋯	⋯	⋯
2^{++}	$J_{4\alpha\beta}^{2^{++}}$	1	⋯	0.036	⋯	⋯	⋯	0.003	⋯	⋯	⋯	⋯	⋯	⋯
0^{-+}	$J_5^{0^{-+}}$	1	0.071	⋯	⋯	0.21	0.69	⋯	⋯	⋯	⋯	⋯	⋯	⋯
	$J_6^{0^{-+}}$	1	0.071	⋯	⋯	0.21	6.2	⋯	⋯	⋯	⋯	⋯	⋯	⋯
0^{--}	$J_7^{0^{--}}$	⋯	⋯	⋯	⋯	⋯	⋯	⋯	1	0.048	0.078	⋯	1.4	⋯
1^{-+}	$J_{8\alpha}^{1^{-+}}$	1	0.071	⋯	⋯	0.78	⋯	0.94	⋯	⋯	⋯	⋯	⋯	⋯
	$J_{9\alpha}^{1^{-+}}$	1	0.071	⋯	⋯	0.78	⋯	8.4	⋯	⋯	⋯	⋯	⋯	⋯
1^{--}	$J_{10\alpha}^{1^{--}}$	⋯	⋯	⋯	⋯	⋯	⋯	⋯	1	0.048	0.078	0.79	1.5	0.43
	$J_{11\alpha}^{1^{--}}$	⋯	⋯	⋯	⋯	⋯	⋯	⋯	1	0.048	0.078	7.1	1.5	0.43

- The **$X(6200)$ and $X(6600)$ can be S-wave $cc\bar{c}\bar{c}$ tetraquarks with $J^{PC}=0^{++}$ or 2^{++}**, while **$X(6900)$ can be a P-wave $cc\bar{c}\bar{c}$ tetraquark with $J^{PC}=0^{-+}$ or 1^{-+};**
- Their quantum numbers can be determined by studying **the relative branching ratios of the di-J/ψ, di-η_c, $J/\psi h_c$, $\eta_c\chi_{c0}$, $\eta_c\chi_{c1}$ channels.**

$bb\bar{b}\bar{b}$ tetraquarks

The $bb\bar{b}\bar{b}$ tetraquarks lie below two bottomonium thresholds:

- $bb\bar{b}\bar{b} \to (b\bar{b}) + (b\bar{b})$: kinematically forbidden.
- $bb\bar{b}\bar{b} \to (bbq) + (\bar{b}\bar{b}\bar{q})$: kinematically forbidden.
- $bb\bar{b}\bar{b} \to (q\bar{b}) + (b\bar{q})$: possible via a heavy quark pair annihilation and a light quark pair creation.
- $bb\bar{b}\bar{b} \to (b\bar{b}) + \gamma$: electromagnetic decay via $b\gamma_\mu\bar{b} \to \gamma$ photon production process.
- These $bb\bar{b}\bar{b}$ states are expected to be very narrow. They are good candidates for compact tetraquarks, if they do exist.



$bb\bar{b}\bar{b}$ decays

Phys. Rev. D 106 (2022), 094019

J^{PC}	Current	Decay channels						
		$\mu^+\mu^-\Upsilon(1S)$	$\mu^+\mu^-\Upsilon(2S)$	$\mu^+\mu^-h_b(1P)$	$\mu^+\mu^-\eta_b(1S)$	$\mu^+\mu^-\eta_b(2S)$	$\mu^+\mu^-\chi_{b0}(1P)$	$\mu^+\mu^-\chi_{b1}(1P)$
0^{++}	$J_1^{0^{++}}$	1	0.42	0.002	⋯	⋯	⋯	⋯
	$J_2^{0^{++}}$	1	0.42	⋯	⋯	⋯	⋯	⋯
1^{+-}	$J_{3\alpha}^{1^{+-}}$	⋯	⋯	⋯	1	0.42	⋯	1×10^{-4}
2^{++}	$J_{4\alpha\beta}^{2^{++}}$	1	0.42	0.002	⋯	⋯	⋯	⋯
0^{-+}	$J_5^{0^{-+}}$	1	0.39	0.090	⋯	⋯	⋯	⋯
	$J_6^{0^{-+}}$	1	0.39	0.090	⋯	⋯	⋯	⋯
0^{--}	$J_7^{0^{--}}$	⋯	⋯	⋯	1	0.42	⋯	0.041
1^{-+}	$J_{8\alpha}^{1^{-+}}$	1	0.43	0.27	⋯	⋯	⋯	⋯
	$J_{9\alpha}^{1^{-+}}$	1	0.43	0.27	⋯	⋯	⋯	⋯
1^{--}	$J_{10\alpha}^{1^{--}}$	⋯	⋯	⋯	1	0.38	1.3	0.070
	$J_{11\alpha}^{1^{--}}$	⋯	⋯	⋯	1	0.38	12	0.070

- The **$bb\bar{b}\bar{b}$ tetraquarks with $J^{PC}=0^{++}/2^{++}/0^{-+}/1^{-+}$** can be searched for in **the $\mu^+\mu^+\Upsilon(1S)$ and $\mu^+\mu^+\Upsilon(2S)$ channels;**
- The **$bb\bar{b}\bar{b}$ tetraquarks with $J^{PC}=0^{+-}/1^{+-}/1^{--}$** can be searched for in the **$\mu^+\mu^+\eta_b(1S)$ and $\mu^+\mu^+\eta_b(2S)$ channels.**

Summary

- We have calculated the mass spectra for the $cc\bar{c}\bar{c}$, $bb\bar{b}\bar{b}$ tetraquark states, and have studied their decay properties.
- Our results suggest that the **X(6200), X(6600) can be interpreted as an S-wave $cc\bar{c}\bar{c}$ tetraquark state with $J^{PC} = 0^{++}$ or 2^{++}**, while **the narrow structure $X(6900)$ to be a P-wave one with $J^{PC} = 0^{-+}$ or 1^{-+}**. The $X(7200)$ may be a P-wave state with $J^{PC} = 1^{-+}$.
- The relative branching ratios of the di-J/ψ, $J/\psi\psi(2S)$, $J/\psi h_c$, di-η_c, $\eta_c\chi_{c0,1}$ channels are helpful to determine their quantum numbers.
- The $bb\bar{b}\bar{b}$ tetraquarks with $J^{PC} = 0^{++}/2^{++}/0^{-+}/1^{-+}$ can be searched for in the $\mu^+\mu^-\Upsilon(1S)$ and $\mu^+\mu^-\Upsilon(2S)$ channels, while those with $J^{PC} = 0^{+-}/1^{+-}/1^{--}$ can be searched for in the $\mu^+\mu^-\eta_b(1S)$ and $\mu^+\mu^-\eta_b(2S)$ channels.

Thank you for your attention!