Study of semileptonic Λ decays: theoretical and experimental view

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Introduction

- **Motivation**:
  - **Theoretical**: a general modular method for the semileptonic hyperon decays
  - **Experimental**: analysis of the semileptonic hyperon decays using modular method to extract decay parameters

    $\Longrightarrow$ will be presented by Shun
- Work is based on:
  - Polarization observables in e^+e^- annihilation to a $B\bar{B}$ pair [PRD 99 (2019) 056008]
  - Helicity analysis for $\Xi^0 \to \Sigma^+(\to p\pi^0)l^-\bar{\nu}_l$ $(l = e^-, \mu^-)$ [EPJ C59 (2009) 27]
- Helicity method allows:
  - Compact calculations of the angular decay distributions
  - Analyze the semileptonic decays of polarized hyperon
  - Take into account the lepton mass effects

    $\Rightarrow$ vector and axial-vector currents
- Presented work is in a progress...

Production process of two spin-$\frac{1}{2}$ baryons

- General framework of the $e^+e^- \to J/\psi \to \Lambda\bar{\Lambda}$ [PRD99 (2019) 056008]
- Spin density matrix of the production process:

$$\rho_{B_1,\bar{B}_2} = \frac{1}{4}\sum_{\mu,\bar{\nu}=0}^{3} C_{\mu\bar{\nu}}(\theta_1)\sigma_{\mu}^{B_1} \otimes \sigma_{\bar{\nu}}^{\bar{B}_2} \qquad C_{\mu\bar{\nu}} = \begin{pmatrix} 1+\alpha_\psi\cos^2\theta_1 & 0 & \beta_\psi\sin\theta_1\cos\theta_1 & 0 \\ 0 & \sin^2\theta_1 & 0 & \gamma_\psi\sin\theta_1\cos\theta_1 \\ -\beta_\psi\sin\theta_1\cos\theta_1 & 0 & \alpha_\psi\sin^2\theta_1 & 0 \\ 0 & -\gamma_\psi\sin\theta_1\cos\theta_1 & 0 & -\alpha_\psi - \cos^2\theta_1 \end{pmatrix}$$

$$\sigma_0^B = \mathbf{1}_2, \sigma_1^\Lambda = \sigma_x, \sigma_2^\Lambda = \sigma_y \text{ and } \sigma_3^\Lambda = \sigma_z \qquad \beta_\psi = \sqrt{1-\alpha_\psi^2}\sin(\Delta\Phi) \qquad \gamma_\psi = \sqrt{1-\alpha_\psi^2}\cos(\Delta\Phi)$$

- Main parameters of $C_{\mu\bar{\nu}}(\theta_1)$:
  θ_Λ - scattering angle of Λ baryon
  $\alpha_\psi \in [-1,+1]$ - baryon angular distribution parameters
  $\Delta\Phi \in [-\pi,+\pi]$ - relative phase between the two transitions

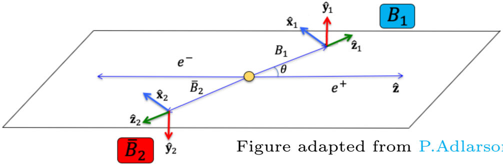

Figure adapted from P.Adlarson's slides

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$$\sigma_0^B = \mathbf{1}_2, \sigma_1^\Lambda = \sigma_x, \sigma_2^\Lambda = \sigma_y \text{ and } \sigma_3^\Lambda = \sigma_z \qquad \beta_\psi = \sqrt{1-\alpha_\psi^2}\sin(\Delta\Phi) \qquad \gamma_\psi = \sqrt{1-\alpha_\psi^2}\cos(\Delta\Phi)$$

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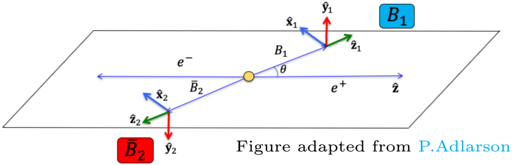
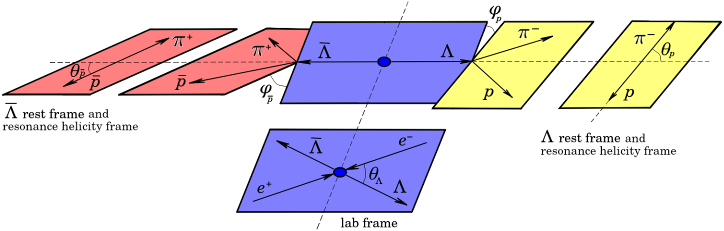


Figure adapted from P.Adlarson's slides

- Production process doesn't depend on the final states. It is the same for:
  - $e^+e^- \to J/\psi \to (\Lambda \to p\pi^-)(\bar{\Lambda} \to \bar{p}\pi^+)$
  - $e^+e^- \to J/\psi \to (\Lambda \to pe^-\bar{\nu}_e)(\bar{\Lambda} \to \bar{p}\pi^+)$

Full hadronic decay chain

- $e^+e^- \to (\Lambda \to p\pi^-)(\bar{\Lambda} \to \bar{p}\pi^+)$ (full determination in [PRD99 (2019) 056008])

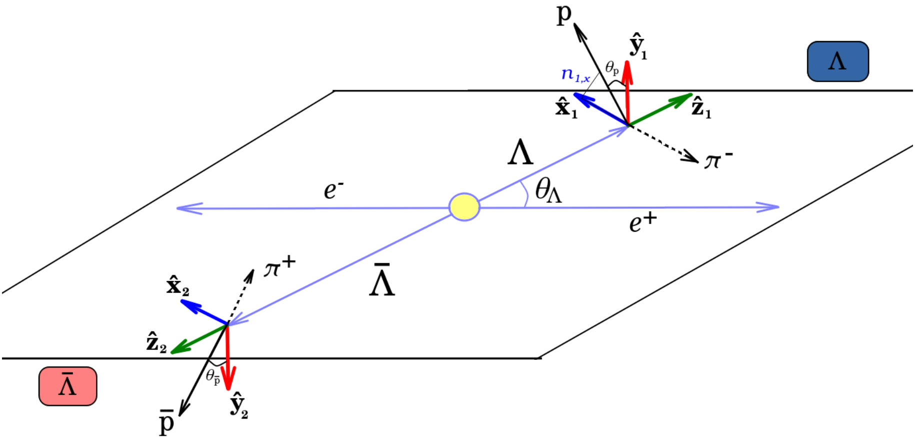


- Decay matrix or transition matrix $a_{\mu\nu}$ for $\{\frac{1}{2} \to \frac{1}{2} + 0\}$

$$\sigma_\mu \to \sum_{\nu=0}^{3} a_{\mu\nu}\sigma_\nu^d$$

- Two helicity amplitudes: $B_{\frac{1}{2}}, B_{-\frac{1}{2}}$
- Main parameters of $a_{\mu\nu}$:
  θ, ϕ - spherical coordinates of the $p/\bar{p}$ momentum in the $\Lambda/\bar{\Lambda}$ helicity frame
  $\alpha_D \in [-1, +1]$ and $\phi_D \in [-\pi, +\pi]$ - decay parameters $(D = \Lambda, \bar{\Lambda})$

Exclusive joint angular distribution (1)



- $\Lambda \to p\pi^-$: $\hat{\mathbf{n}}_1 \to (\cos\theta_p, \phi_p)$: α_Λ
- $\bar{\Lambda} \to \bar{p}\pi^+$: $\hat{\mathbf{n}}_2 \to (\cos\theta_{\bar{p}}, \phi_{\bar{p}})$: $\alpha_{\bar{\Lambda}}$

Exclusive joint angular distribution (2)

$$\mathrm{Tr}\rho_{p\bar{p}} \propto \mathcal{W}(\boldsymbol{\xi};\boldsymbol{\omega}) = \sum_{\mu,\bar{\nu}=0}^{3} C_{\mu\bar{\nu}} a^{\Lambda}_{\mu 0} a^{\bar{\Lambda}}_{\bar{\nu}0}$$

- $C_{\mu\bar{\nu}}(\theta_\Lambda;\alpha_\psi,\Delta\Phi)$
- $a_{\mu 0}$ matrices for $1/2 \to 1/2+0$ decays $\Leftrightarrow a^{\Lambda}_{\mu 0} \equiv a_{\mu 0}(\theta_p,\varphi_p;\boxed{\alpha_\Lambda})$
- $a_{\bar{\nu}0}$ matrices for $1/2 \to 1/2+0$ decays $\Leftrightarrow a^{\bar{\Lambda}}_{\bar{\nu}0} \equiv a_{\bar{\nu}0}(\theta_{\bar{p}},\varphi_{\bar{p}};\boxed{\alpha_{\bar{\Lambda}}})$

  - $\boldsymbol{\xi} \equiv (\theta_\Lambda,\theta_p,\varphi_p,\theta_{\bar{p}},\varphi_{\bar{p}})$ $\longrightarrow$ 5D PhSp
  - $\boldsymbol{\omega} \equiv (\alpha_\psi,\Delta\Phi,\boxed{\alpha_\Lambda},\boxed{\alpha_{\bar{\Lambda}}})$

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    - $\boldsymbol{\xi} \equiv (\theta_\Lambda,\theta_p,\varphi_p,\theta_{\bar{p}},\varphi_{\bar{p}}) \longrightarrow$ 5D PhSp
    - $\boldsymbol{\omega} \equiv (\alpha_\psi,\Delta\Phi,\boxed{\alpha_\Lambda},\boxed{\alpha_{\bar{\Lambda}}})$

$$\mathrm{d}\Gamma \propto \mathcal{W}(\boldsymbol{\xi};\boldsymbol{\alpha_\psi},\boldsymbol{\Delta\Phi},\boldsymbol{\alpha_\Lambda},\boldsymbol{\alpha_{\bar{\Lambda}}}) =$$

$$\underbrace{\boxed{1+\boldsymbol{\alpha_\psi}\cos^2\theta_\Lambda}}_{\text{Cross section}}$$

$$\underbrace{\boxed{\begin{aligned}&+\boldsymbol{\alpha_\Lambda}\boldsymbol{\alpha_{\bar{\Lambda}}}\left(\sin^2\theta_\Lambda(n_{1,x}n_{2,x}-\boldsymbol{\alpha_\psi}n_{1,y}n_{2,y})+(\cos^2\theta_\Lambda+\boldsymbol{\alpha_\psi})n_{1,z}n_{2,z}\right)\\&+\boldsymbol{\alpha_\Lambda}\boldsymbol{\alpha_{\bar{\Lambda}}}\sqrt{1-\boldsymbol{\alpha_\psi}^2}\cos(\boldsymbol{\Delta\Phi})\sin\theta_\Lambda\cos\theta_\Lambda(n_{1,x}n_{2,z}+n_{1,z}n_{2,x})\end{aligned}}}_{\text{Spin correlations}}$$

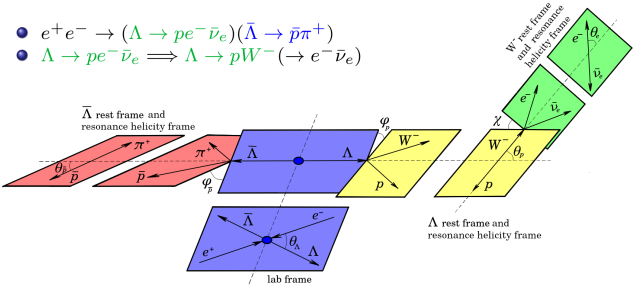
$$\underbrace{\boxed{+\sqrt{1-\boldsymbol{\alpha_\psi}^2}\sin(\boldsymbol{\Delta\Phi})\sin\theta_\Lambda\cos\theta_\Lambda\left(\boldsymbol{\alpha_\Lambda}n_{1,y}+\boldsymbol{\alpha_{\bar{\Lambda}}}n_{2,y}\right)}}_{\text{Polarization}}$$

- $\Delta\Phi \neq 0 \Rightarrow$ independent determination of α_Λ and $\alpha_{\bar{\Lambda}}$ [PLB772(2017)16]
- BESIII measurement [Nature Phys.15(2019)631]:

$\alpha_\psi = 0.461 \pm 0.006 \pm 0.007$	$\alpha_- = 0.750 \pm 0.009 \pm 0.004$
$\Delta\Phi = (42.4 \pm 0.6 \pm 0.5)^\circ$	$\alpha_+ = -0.758 \pm 0.010 \pm 0.007$

Semileptonic-hadronic decay chain (1)

- $e^+e^- \to (\Lambda \to pe^-\bar{\nu}_e)(\bar{\Lambda} \to \bar{p}\pi^+)$
- $\Lambda \to pe^-\bar{\nu}_e \Longrightarrow \Lambda \to pW^-(\to e^-\bar{\nu}_e)$



- Decay matrix or transition matrix $b_{\mu\nu}$ for $\{\frac{1}{2} \to \frac{1}{2} + \{0, \pm 1\}\}$

$$\sigma_\mu \to \sum_{\nu=0}^{3} b_{\mu\nu}\sigma_\nu^{d-W}$$

- Four helicity amplitudes: $H_{\frac{1}{2}0}, H_{-\frac{1}{2}0}, H_{\frac{1}{2}1}, H_{-\frac{1}{2}-1}$
- Main parameters of $b_{\mu\nu}$: $\Omega = \{\phi, \theta, 0\}$ and $\Omega' = \{\chi, \theta_l, 0\}$
  $q^2 \in (m_e^2, (M_\Lambda - M_p)^2)$; $\alpha_D^{sl}(\theta_l, q^2)$ and $\phi_D^{sl}(\theta_l, q^2)$ - decay parameters ($D = \Lambda, \bar{\Lambda}$)

Semileptonic Λ decay

- Momenta and masses: $\Lambda(p_1, M_1) \to p(p_2, M_2) + e^-(p_e, m_e) + \bar{\nu}_e(p_{\bar{\nu}_e}, 0)$
- FF for the weak current-induced baryonic $1/2^+ \to 1/2^+$ transitions [EPJ C59 (2009) 27]:

$$\langle p(p_2)|J_\mu^{V+A}|\Lambda(p_1)\rangle = \bar{u}(p_2)\left[\gamma_\mu\left(F_1^V(q^2) + F_1^A(q^2)\gamma_5\right) + \frac{i\sigma_{\mu\nu}q^\nu}{M_1}\left(F_2^V(q^2) + F_2^A(q^2)\gamma_5\right) + \frac{q_\mu}{M_1}\left(F_3^V(q^2) + F_3^A(q^2)\gamma_5\right)\right]u(p_1)$$

where $q_\mu = (p_1 - p_2)_\mu$

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where $q_\mu = (p_1 - p_2)_\mu$

- For $\Lambda \to pe^-\bar{\nu}_e$ at $\mathcal{O}(\frac{m_e^2}{2q^2}) \to 0 \Rightarrow F_3^{V,A}(q^2) \to 0$
- $H_{\lambda_2\lambda_W} = (H^V_{\lambda_2\lambda_W} + H^A_{\lambda_2\lambda_W})$ with $(\lambda_2 = \pm 1/2; \lambda_W = 0, \pm 1)$: $H^{V,A}_{\lambda_2\lambda_W} \equiv H^{V,A}_{\lambda_2\lambda_W}(F^{V,A}_{1,2}(q^2))$

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vector

$$H^V_{\frac{1}{2}1} = \sqrt{2M_-}\left(-F_1^V - \frac{M_1 + M_2}{M_1}F_2^V\right),$$

$$H^V_{\frac{1}{2}0} = \frac{\sqrt{M_-}}{\sqrt{q^2}}\left((M_1 + M_2)F_1^V + \frac{q^2}{M_1}F_2^V\right),$$

axial-vector

$$H^A_{\frac{1}{2}1} = \sqrt{2M_+}\left(F_1^A - \frac{M_1 - M_2}{M_1}F_2^A\right),$$

$$H^A_{\frac{1}{2}0} = \frac{\sqrt{M_+}}{\sqrt{q^2}}\left(-(M_1 - M_2)F_1^A + \frac{q^2}{M_1}F_2^A\right).$$

where $M_\pm = (M_1 \pm M_2)^2 - q^2$; $\quad H^{V,A}_{-\lambda_2,-\lambda_W} = \pm H^{V,A}_{\lambda_2\lambda_W}$

Form factors

$$F_i^{V,A}(q^2) = F_i^{V,A}(0)\prod_{n=0}^{n_i}\frac{1}{1-\frac{q^2}{m_{V,A}^2+n\alpha'^{-1}}} \approx F_i^{V,A}(0)\left(1+q^2\sum_{n=0}^{n_i}\frac{1}{m_{V,A}^2+n\alpha'^{-1}}\right) \equiv F_i^{V,A}(0)c_i^{V,A}(q^2)$$

	$F_i^{V,A}(0)(\Lambda \to p)$	$m_{V,A}$	α' [GeV^{-2}]	n_i
$F_1^V(q^2)$	$-\sqrt{\frac{3}{2}}$ [1]	$m_{K^*(892)^0}$ $(J^P = 1^-)$	0.9	$n_1 = 1$
$F_2^V(q^2)$	$\frac{M_\Lambda \mu_p}{2M_p}F_1^V(0)$ [2]			$n_2 = 2$
$F_3^V(q^2)$	0 [4]	-		$n_3 = 2$
$F_1^A(q^2)$	$0.719F_1^V(0)$ [3]	$m_{K^*(1270)^0}$ $(J^P = 1^+)$		$n_1 = 1$
$F_2^A(q^2)$	0 [4]	–		$n_2 = 2$
$F_3^A(q^2)$	$\frac{M_\Lambda(M_\Lambda+M_p)}{(m_{K^-})^2}F_1^A(0)$ [4]	m_K $(J^P = 0^-)$		$n_3 = 2$

- [1] [PR135(1964)B1483], [PRL13(1964)264]
- [2] $\mu_p = 1.793$ [Lect.NotesPhys.222(1985)1], [Ann.Rev.Nucl.Part.Sci.53(2003)39], [JHEP0807(2008)132]
- [3] [PRD41(1990)780]
- [4] Vanish in the $SU(3)$ symmetry limit; Goldberger-Treiman relation [PR110(1958)1178], [PR111(1958)354]

Intermediate step

- $\alpha_{\Lambda}^{sl}(\theta_l, q^2) \Rightarrow \{\alpha, \alpha', \alpha'', \beta_{1,2}, \gamma_{1,2}\}(q^2)$ and g_{av}^{Λ}, g_w^{Λ}
- α, α', α'', $\beta_{1,2}$ and $\gamma_{1,2} \in [-1, +1]$
- Introduce the intermediate parameters:

$$\begin{aligned}
\text{normalization} \quad n &= |H_{\frac{1}{2}1}|^2 + |H_{-\frac{1}{2}-1}|^2 + 2(|H_{-\frac{1}{2}0}|^2 + |H_{\frac{1}{2}0}|^2),\\
\alpha &= |H_{\frac{1}{2}1}|^2 - |H_{-\frac{1}{2}-1}|^2 + 2(|H_{-\frac{1}{2}0}|^2 - |H_{\frac{1}{2}0}|^2),\\
\alpha' &= |H_{\frac{1}{2}1}|^2 + |H_{-\frac{1}{2}-1}|^2 - 2(|H_{-\frac{1}{2}0}|^2 + |H_{\frac{1}{2}0}|^2),\\
\alpha'' &= |H_{\frac{1}{2}1}|^2 - |H_{-\frac{1}{2}-1}|^2 - 2(|H_{-\frac{1}{2}0}|^2 - |H_{\frac{1}{2}0}|^2),\\
\beta_{1,2} &= 2(\Im(H^*_{-\frac{1}{2}0} H_{-\frac{1}{2}-1}) \pm \Im(H^*_{\frac{1}{2}1} H_{\frac{1}{2}0})),\\
\gamma_{1,2} &= 2(\Re(H^*_{-\frac{1}{2}0} H_{-\frac{1}{2}-1}) \pm \Re(H^*_{\frac{1}{2}1} H_{\frac{1}{2}0})),
\end{aligned}$$

where $\beta_{1,2} = \frac{1}{2}\sqrt{n^2 - \alpha^2 - (\alpha')^2 + (\alpha'')^2}\sin\phi_{1,2}$ and $\gamma_{1,2} = \frac{1}{2}\sqrt{n^2 - \alpha^2 - (\alpha')^2 + (\alpha'')^2}\cos\phi_{1,2}$

- $\alpha^2 + (\alpha')^2 - (\alpha'')^2 + 2\sum_{i=1}^{2}(\gamma_i^2 + \beta_i^2) = n^2$
- Main parameters to describe semileptonic hyperon decays are:

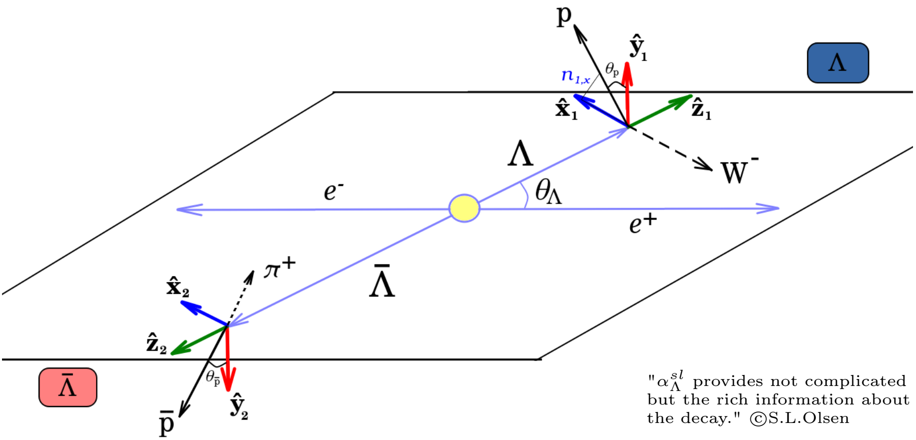
  - $F_1^V(0)$, $F_2^V(0)$, $F_1^A(0)$

  or

  - $g_{av}^D(0) = \dfrac{F_1^A(0)}{F_1^V(0)}$, $\quad g_w^D(0) = \dfrac{F_2^V(0)}{F_1^V(0)}$

- Last measurement of g_{av} and g_w in $\Lambda \to pe^-\bar{\nu}_e$ by E-555 (Fermilab) [PRD41 (1990) 780]
  - $g_{av} = 0.731 \pm 0.016$ and $g_w = 0.15 \pm 0.30$
  - $g_{av} = 0.719 \pm 0.016$ with constraint $g_w \approx 0.97$ (CVC)

Exclusive joint angular distribution (1)



"α_Λ^{sl} provides not complicated but the rich information about the decay." ©S.L.Olsen

- $\Lambda \to pW^-$: $\hat{\mathbf{n}}_1 \to (\cos\theta_p, \phi_p)$: $\alpha_\Lambda^{sl}(W^- \to e^-\bar{\nu}_e) \equiv \alpha_\Lambda^{sl}$
  - $W^- \to e^-\bar{\nu}_e$: (θ_e, χ, q^2) : $g_{av}^\Lambda, g_w^\Lambda$
- $\bar{\Lambda} \to \bar{p}\pi^+$: $\hat{\mathbf{n}}_2 \to (\cos\theta_{\bar{p}}, \phi_{\bar{p}})$: $\alpha_{\bar{\Lambda}}$

Exclusive joint angular distribution (2)

$$\mathrm{Tr}\rho_{pW\bar{p}} \propto \mathcal{W}(\boldsymbol{\xi};\boldsymbol{\omega}) = \sum_{\mu,\bar{\nu}=0}^{3} C_{\mu\bar{\nu}} b^{\Lambda}_{\mu 0} a^{\bar{\Lambda}}_{\bar{\nu}0}$$

- $C_{\mu\bar{\nu}}(\theta_\Lambda;\alpha_\psi,\Delta\Phi)$
- $b_{\mu 0}$ matrices for $1/2 \to 1/2 + \{0,\pm 1\}$ decays $\Leftrightarrow b^{\Lambda}_{\mu 0} \equiv b_{\mu 0}(\theta_p,\varphi_p,\theta_e,\chi,q^2;\boxed{g^{\Lambda}_{av},g^{\Lambda}_{w}})$
- $a_{\bar{\nu}0}$ matrices for $1/2 \to 1/2+0$ decays $\Leftrightarrow a^{\bar{\Lambda}}_{\bar{\nu}0} \equiv a_{\bar{\nu}0}(\theta_{\bar{p}},\varphi_{\bar{p}};\boxed{\alpha_{\bar{\Lambda}}})$
  - $\boldsymbol{\xi} \equiv (\theta_\Lambda,\theta_p,\varphi_p,\theta_e,\chi,q^2,\theta_{\bar{p}},\varphi_{\bar{p}}) \longrightarrow$ 8D PhSp
  - $\boldsymbol{\omega} \equiv (\alpha_\psi,\Delta\Phi,\boxed{g^{\Lambda}_{av},g^{\Lambda}_{w}},\boxed{\alpha_{\bar{\Lambda}}})$

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  - $\boldsymbol{\xi} \equiv (\theta_\Lambda,\theta_p,\varphi_p,\theta_e,\chi,q^2,\theta_{\bar{p}},\varphi_{\bar{p}}) \longrightarrow$ 8D PhSp
  - $\boldsymbol{\omega} \equiv (\alpha_\psi,\Delta\Phi,\boxed{g^{\Lambda}_{av},g^{\Lambda}_{w}},\boxed{\alpha_{\bar{\Lambda}}})$

- $\boldsymbol{\xi}'$: $(\cos\theta_\Lambda,\hat{\mathbf{n}}_1,\hat{\mathbf{n}}_2) \leftarrow$ 5D PhSp

$$\mathrm{d}\Gamma \propto \mathcal{W}(\boldsymbol{\xi}';\boldsymbol{\alpha}_\psi,\boldsymbol{\Delta\Phi},\boldsymbol{\alpha}^{sl}_{\Lambda},\boldsymbol{\alpha}_{\bar{\Lambda}}) =$$

$$\boxed{1+\boldsymbol{\alpha}_\psi\cos^2\theta_\Lambda} \quad \text{Cross section}$$

$$\boxed{\begin{aligned}&+\boldsymbol{\alpha}^{sl}_{\Lambda}\boldsymbol{\alpha}_{\bar{\Lambda}}\left(\sin^2\theta_\Lambda(n_{1,x}n_{2,x}-\boldsymbol{\alpha}_\psi n_{1,y}n_{2,y})+(\cos^2\theta_\Lambda+\boldsymbol{\alpha}_\psi)n_{1,z}n_{2,z}\right)\\&+\boldsymbol{\alpha}^{sl}_{\Lambda}\boldsymbol{\alpha}_{\bar{\Lambda}}\sqrt{1-\boldsymbol{\alpha}_\psi{}^2}\cos(\boldsymbol{\Delta\Phi})\sin\theta_\Lambda\cos\theta_\Lambda(n_{1,x}n_{2,z}+n_{1,z}n_{2,x})\end{aligned}} \quad \text{Spin correlations}$$

$$\boxed{+\sqrt{1-\boldsymbol{\alpha}_\psi{}^2}\sin(\boldsymbol{\Delta\Phi})\sin\theta_\Lambda\cos\theta_\Lambda\left(\boldsymbol{\alpha}^{sl}_{\Lambda}n_{1,y}+\boldsymbol{\alpha}_{\bar{\Lambda}}n_{2,y}\right)} \quad \text{Polarization}$$

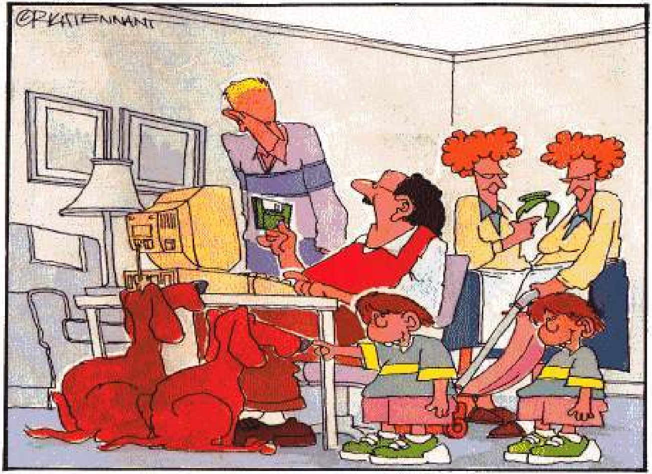
- $\Delta\Phi \neq 0 \Rightarrow$ independent determination of α^{sl}_{Λ} and $\alpha_{\bar{\Lambda}}$
- Same expression for $e^+e^- \to (\Lambda \to p\pi^-)(\bar{\Lambda} \to \bar{p}\pi^+)$ [PLB772(2017)16]: $\alpha^{sl}_{\Lambda} \Leftrightarrow \alpha_\Lambda$
- Possible measurement of g_{av} and g_w using BESIII data

Interim summary and outline

- Determination of helicity formalism to describe semileptonic-hadronic decay chain is in a progress...
- Introduction of a general modular method for the semileptonic decays will allow to describe
  - semileptonic baryon/hyperon decays
  - semileptonic cascade decays
- Verification of the formalism using toy MC sample
- Test the formalism using BESIII MC and data
  $\Longrightarrow$ will be presented by Shun
- **Next steps**:
  - Preparation of phenomenology paper

Thank you for your attention!

Backups



" I ALWAYS BACK UP EVERYTHING."

Decay chain: $\frac{1}{2} \to \frac{1}{2} + 0$

[PRD99(2019)056008]

- Relation between helicity amplitudes and decay parameters

$$\text{normalization } |A_S|^2 + |A_P|^2 = |B_{-\frac{1}{2}}|^2 + |B_{\frac{1}{2}}|^2 = 1,$$
$$\alpha_D = -2\Re(A_S^* A_P) = |B_{1/2}|^2 - |B_{-1/2}|^2,$$
$$\beta_D = -2\Im(A_S^* A_P) = 2\Im(B_{1/2} B^*_{-1/2}),$$
$$\gamma_D = |A_S^*|^2 - |A_P|^2 = 2\Re(B_{1/2} B^*_{-1/2}),$$
$$\text{where } \beta_D = \sqrt{1-\alpha_D^2}\sin\varphi_D \text{ and } \gamma_D = \sqrt{1-\alpha_D^2}\cos\varphi_D$$

- Non-zero elements of the decay matrix $a_{\mu\nu}$:

$$\boxed{a_{00} = 1,} \qquad a_{21} = \beta_D \cos\varphi + \gamma_D \cos\theta \sin\varphi,$$
$$a_{03} = \alpha_D, \qquad a_{22} = \gamma_D \cos\varphi - \beta_D \cos\theta \sin\varphi,$$
$$\boxed{a_{10} = \alpha_D \cos\varphi \sin\theta,} \qquad a_{23} = \sin\theta \sin\varphi,$$
$$a_{11} = \gamma_D \cos\theta \cos\varphi - \beta_D \sin\varphi, \qquad \boxed{a_{30} = \alpha_D \cos\theta,}$$
$$a_{12} = -\beta_D \cos\theta \cos\varphi - \gamma_D \sin\varphi, \qquad a_{31} = -\gamma_D \sin\theta,$$
$$a_{13} = \sin\theta \cos\varphi, \qquad a_{32} = \beta_D \sin\theta,$$
$$\boxed{a_{20} = \alpha_D \sin\theta \sin\varphi,} \qquad a_{33} = \cos\theta$$

- Main parameters: $\theta \equiv \theta_{p/\bar{p}}$, $\varphi \equiv \varphi_{p/\bar{p}}$, $\alpha_D \equiv \alpha_{\Lambda/\bar{\Lambda}}$, $\varphi_D \equiv \varphi_{\Lambda/\bar{\Lambda}}$

Decay chain: $\frac{1}{2} \to \frac{1}{2} + \{0, \pm 1\}$

- Relation between helicity amplitudes and decay parameters

$$\text{normalization} \;\; \frac{1}{4}(1-\cos\theta_l)^2|H_{\frac{1}{2}1}|^2 + \frac{1}{4}(1+\cos\theta_l)^2|H_{-\frac{1}{2}-1}|^2 + \frac{1}{2}\sin^2\theta_l(|H_{-\frac{1}{2}0}|^2 + |H_{\frac{1}{2}0}|^2) = 1,$$

$$\alpha_D^{sl} = \frac{1}{4}(1-\cos\theta_l)^2|H_{\frac{1}{2}1}|^2 - \frac{1}{4}(1+\cos\theta_l)^2|H_{-\frac{1}{2}-1}|^2 + \frac{1}{2}\sin^2\theta_l(|H_{-\frac{1}{2}0}|^2 - |H_{\frac{1}{2}0}|^2),$$

$$\beta_D^{sl} = \frac{1}{\sqrt{2}}\sin\theta_l\,((1+\cos\theta_l)\Im(H^*_{-\frac{1}{2}0}H_{-\frac{1}{2}-1}) + (1-\cos\theta_l)\Im(H^*_{\frac{1}{2}1}H_{\frac{1}{2}0})),$$

$$\gamma_D^{sl} = \frac{1}{\sqrt{2}}\sin\theta_l\,((1+\cos\theta_l)\Re(H^*_{-\frac{1}{2}0}H_{-\frac{1}{2}-1}) + (1-\cos\theta_l)\Re(H^*_{\frac{1}{2}1}H_{\frac{1}{2}0}))$$

$$\text{where } \beta_D^{sl} = \sqrt{1-(\alpha_D^{sl})^2}\sin\phi_D^{sl} \text{ and } \gamma_D^{sl} = \sqrt{1-(\alpha_D^{sl})^2}\cos\phi_D^{sl}$$

- Non-zero elements of the decay matrix $b_{\mu\nu}$:

$$\begin{aligned}
b_{00} &= \sigma_D^{sl}, \\
b_{03} &= \alpha_D^{sl}, \\
b_{10} &= \alpha_D^{sl}\cos\phi_p\sin\theta_p, \\
b_{11} &= -(\gamma_D^{sl}\cos\chi + \beta_D^{sl}\sin\chi)\cos\theta_p\cos\phi_p \\
&\quad +(\gamma_D^{sl}\sin\chi - \beta_D^{sl}\cos\chi)\sin\phi_p, \\
b_{12} &= \quad (\gamma_D^{sl}\sin\chi - \beta_D^{sl}\cos\chi)\cos\theta_p\cos\phi_p \\
&\quad +(\gamma_D^{sl}\cos\chi + \beta_D^{sl}\sin\chi)\sin\phi_p, \\
b_{13} &= \sigma_D^{sl}\sin\theta_p\cos\phi_p, \\
b_{20} &= \alpha_D^{sl}\sin\theta_p\sin\phi_p,
\end{aligned}$$

$$\begin{aligned}
b_{21} &= -(\gamma_D^{sl}\cos\chi + \beta_D^{sl}\sin\chi)\cos\theta_p\sin\phi_p \\
&\quad -(\gamma_D^{sl}\sin\chi - \beta_D^{sl}\cos\chi)\cos\phi_p, \\
b_{22} &= \quad (\gamma_D^{sl}\sin\chi - \beta_D^{sl}\cos\chi)\cos\theta_p\sin\phi_p \\
&\quad -(\gamma_D^{sl}\cos\chi + \beta_D^{sl}\sin\chi)\cos\phi_p, \\
b_{23} &= \sigma_D^{sl}\sin\theta_p\sin\phi_p, \\
b_{30} &= \alpha_D^{sl}\cos\theta_p, \\
b_{31} &= \quad (\gamma_D^{sl}\cos\chi + \beta_D^{sl}\sin\chi)\sin\theta_p, \\
b_{32} &= -(\gamma_D^{sl}\sin\chi - \beta_D^{sl}\cos\chi)\sin\theta_p, \\
b_{33} &= \sigma_D^{sl}\cos\theta_p.
\end{aligned}$$

- Main parameters:
  $\sigma_D^{sl} \equiv \sigma_D^{sl}(\theta_l, q^2)$, $\alpha_D^{sl} \equiv \alpha_D^{sl}(\theta_l, q^2)$, $\beta_D^{sl} \equiv \beta_D^{sl}(\theta_l, q^2)$, $\gamma_D^{sl} \equiv \gamma_D^{sl}(\theta_l, q^2)$
- Each element of $b_{\mu\nu}$ is multiplied by $q^2 p$ where $p = \sqrt{M_+(q^2)M_-(q^2)}/(2M_1)$

Intermediate step (2)

- Relations between intermediate and decay parameters:

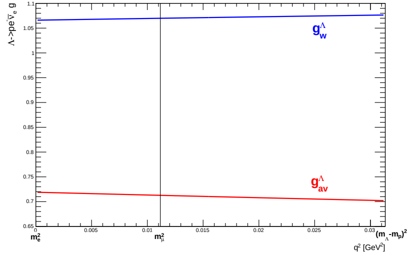
$$n = ((M_-M_+)^2 - q^4)(1 + (g_{av}^D(q^2))^2) + q^2\left(4M_1M_2((g_{av}^D(q^2))^2 - 1) + \frac{q^2}{M_1^2}Q_-((g_w^D(q^2))^2(M_+^2 + q^2) + 4g_w^D(q^2)M_+)\right)$$

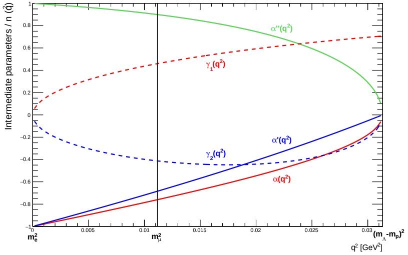
$$\alpha = 2\sqrt{Q_-Q_+}\left[g_{av}^D(q^2)(q^2 - M_-M_+) + 2g_{av}^D(q^2)g_w^D(q^2)q^2\frac{M_2}{M_1}\right]$$

$$\alpha' = Q_-Q_+\left[-(1 + (g_{av}^D(q^2))^2) + (g_w^D(q^2))^2\frac{q^2}{M_1^2}\right]$$

$$\alpha'' = 2\sqrt{Q_-Q_+}\left[g_{av}^D(q^2)(q^2 + M_-M_+) + 2g_{av}^D(q^2)g_w^D(q^2)q^2\right]$$

where $M_- = M_1 - M_2$ and $M_+ = M_1 + M_2$ and $Q_\pm = M_\pm^2 - q^2$





Intermediate step (3)

- Non-zero elements of the decay matrix $b_{\mu\nu}$:

$$
\begin{aligned}
&\boxed{b_{00} = 1,} && b_{21} = \mp A\cos\theta_p \sin\phi_p \mp B\cos\phi_p,\\
&b_{03} = a_D^{sl}, && b_{22} = \pm B\cos\theta_p \sin\phi_p \mp A\cos\phi_p,\\
&\boxed{b_{10} = a_D^{sl}\cos\phi_p \sin\theta_p,} && b_{23} = \sin\theta_p \sin\phi_p,\\
&b_{11} = \mp A\cos\theta_p\cos\phi_p \pm B\sin\phi_p, && \boxed{b_{30} = a_D^{sl}\cos\theta_p,}\\
&b_{12} = \pm B\cos\theta_p\cos\phi_p \pm A\sin\phi_p, && b_{31} = \pm A\sin\theta_p,\\
&b_{13} = \sin\theta_p\cos\phi_p, && b_{32} = \mp B\sin\theta_p,\\
&\boxed{b_{20} = a_D^{sl}\sin\theta_p\sin\phi_p,} && b_{33} = \cos\theta_p,
\end{aligned}
$$

$$
\text{where}\quad a_D^{sl} = \frac{\alpha_D^{sl}(\theta_l, q^2)}{\sigma_D^{sl}(\theta_l, q^2)} = \frac{\alpha + \alpha''\cos^2\theta_l \mp (n+\alpha')\cos\theta_l}{n + \alpha'\cos^2\theta_l \mp (\alpha+\alpha'')\cos\theta_l},
$$

$$
A = \frac{1}{2\sqrt{2}}\frac{\sin\theta_l}{\sigma_D^{sl}(\theta_l, q^2)}[\cos\chi(\gamma_1 \pm \cos\theta_l\gamma_2) + \sin\chi(\beta_1 \pm \cos\theta_l\beta_2)],
$$

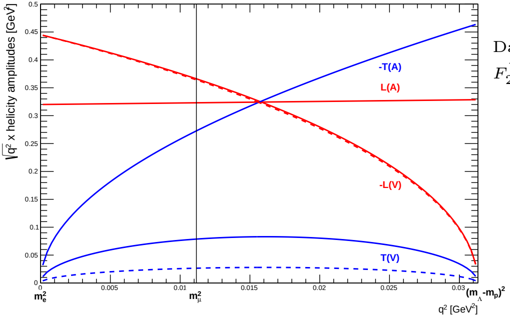
$$
B = \frac{1}{2\sqrt{2}}\frac{\sin\theta_l}{\sigma_D^{sl}(\theta_l, q^2)}[\sin\chi(\gamma_1 \pm \cos\theta_l\gamma_2) - \cos\chi(\beta_1 \pm \cos\theta_l\beta_2)].
$$

Form factors

$$F_i^{V,A}(q^2) = F_i^{V,A}(0)\prod_{n=0}^{n_i}\frac{1}{1-\frac{q^2}{m_{V,A}^2+n\alpha'^{-1}}} \approx F_i^{V,A}(0)\left(1+q^2\sum_{n=0}^{n_i}\frac{1}{m_{V,A}^2+n\alpha'^{-1}}\right)$$

$\mathrm{T(V,A)}:=\sqrt{q^2}H^{V,A}_{\frac{1}{2}1}$

$\mathrm{L(V,A)}:=\sqrt{q^2}H^{V,A}_{\frac{1}{2}0}$



Dashed lines:
$F_2^V(q^2)$ are switched off

- If $q^2_{\min} = m_e^2 \approx 0 \Rightarrow H^V_{\frac{1}{2}0}$ and $H^A_{\frac{1}{2}0}$ are dominated
- If $q^2_{\max} = (M_\Lambda - M_p)^2 \Rightarrow H^A_{\frac{1}{2}1} = -\sqrt{2}H^A_{\frac{1}{2}0}$ are dominated

Helicity amplitudes of the lepton pair $h^l_{\lambda_l \lambda_\nu}$

- Lepton and antineutrino spinors

$$\bar{u}_{l^-}(\mp\frac{1}{2}, p_{l^-}) = \sqrt{E_l + m_l}\left(\chi^\dagger_\mp, \frac{\pm|\vec{p}_{l^-}|}{E_l + m_l}\chi^\dagger_\mp\right),$$

$$v_{\bar{\nu}}(\frac{1}{2}) = \sqrt{E_\nu}\begin{pmatrix}\chi_+ \\ -\chi_+\end{pmatrix},$$

where $\chi_+ = \begin{pmatrix}1\\0\end{pmatrix}$ and $\chi_- = \begin{pmatrix}0\\1\end{pmatrix}$ are Pauli two-spinors

- SM form of the lepton current ($\lambda_W = \lambda_{l^-} - \lambda_{\bar{\nu}}$)

$$h^l_{\lambda_{l^-}=\mp 1/2, \lambda_{\bar{\nu}}=1/2} = \bar{u}_{l^-}(\mp\frac{1}{2})\gamma^\mu(1+\gamma_5)v_{\bar{\nu}}(\frac{1}{2})\begin{Bmatrix}\epsilon_\mu(-1)\\ \epsilon_\mu(0)\end{Bmatrix}$$

where $\epsilon^\mu(0) = (0; 0, 0, 1)$ and $\epsilon^\mu(\mp 1) = (0; \mp 1, -i, 0)/\sqrt{2}$

- Moduli squared of the helicity amplitudes [EPJ C59 (2009) 27]

$$\text{nonflip}(\lambda_W = \mp 1) : |h^l_{\lambda_l=\mp\frac{1}{2}, \lambda_\nu=\pm\frac{1}{2}}|^2 = 8(q^2 - m_l^2),$$

$$\text{flip}(\lambda_W = 0) : |h^l_{\lambda_l=\pm\frac{1}{2}, \lambda_\nu=\pm\frac{1}{2}}|^2 = 8\frac{m_l^2}{2q^2}(q^2 - m_l^2)$$

- Upper and lower signs refer to the configurations $(l^-, \bar{\nu}_l)$ $(\lambda_\nu = 1/2)$ and (l^+, ν_l) $(\lambda_\nu = -1/2)$, respectively
- In case of the e-mode only nonflip transition remains under assumption $\frac{m_e^2}{2q^2} \to 0$