# Two-photon transitions of charmonia on the light front

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2022 年 7 月 18 日，安徽合肥

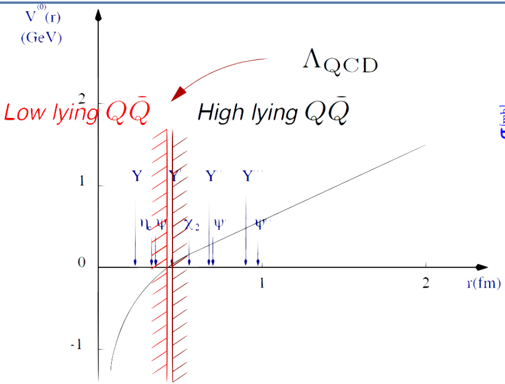
# Outline

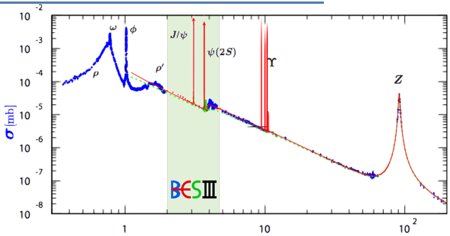
- Physics of two-photon transition of charmonium
- Light front dynamics and basis light-front quantization
- Numerical results: two-photon width and transition form factors
- Summary

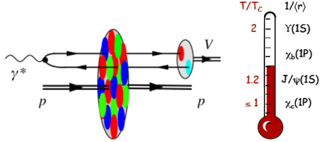
Based on: YL, M. Li (李枚键) and J.P. Vary, Phys. Rev. D letter **105** (2022); arXiv:2111.14178 [hep-ph]

# "A golden system to study strong interactions", "the H-atom of QCD"


丁肇中的十一月革命




Tremendous applications in ee, ep, eA, pp, AA collisions

- Theoretically a hard problem: multiscale, multi-physics

$$\Lambda_{\rm QCD} \lesssim \alpha_s^2 m_c < \alpha_s m_c < m_c$$

- Physically a simple system: nonrelativistic ($v_c \ll 1$), perturbative ($\alpha_s \ll 1$)

$$v_c^2 \sim 0.3,\ \alpha_s(m_c) \sim (0.3-0.6)\ (?)$$

- Potential model, pNRQCD, Lattice QCD

[Brambilla '14]

# Two-photon transitions of charmonia

## A clean & important probe to hadron structures

[Reviews: Berger '87]

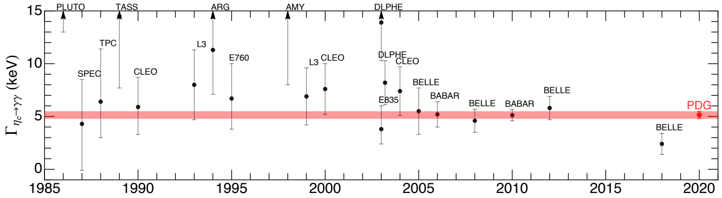
- Decay $H \to \gamma + \gamma$: goden channel for hadron identification
  - *Selection rules: P, C, angular momentum, gauge symmetry, ...*
- Exclusive photoproductions $\gamma + \gamma \to H$: channel of discovery
  - *pQCD factorization at large* $Q^2$, [Lepage '80 & Chernyak '84]

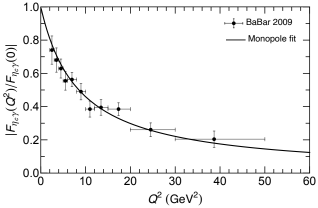
$$F_{\gamma\eta_c}(Q^2) = \int_0^1 \mathrm{d}x\, T_H(x, Q^2) \phi_{\eta_c}(x; \mu),$$

## Experimental measurements

[Review of particle physics 2020]

- Diphoton width: extensive measurements for $\eta_c, \eta_c', \chi_{c0}, \chi_{c2}$;
- Transition form factors: $F_{\eta_c\gamma}(Q^2)$ by BABAR 2010; $F_{\chi_{cJ}\gamma}(Q^2)$ by Belle 2017 with limited statistics

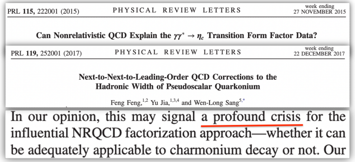


# A crisis in theories for charmonium?

## Status of theoretical predictions

- Potential model: large relativistic corrections [e.g. Babiarz '19]
- NRQCD: $10\sigma$ discrepancy at NNLO -- a crisis for NRQCD? [Feng PRL '15&'17]
- Lattice QCD: huge discrepancy -- many have given up hope in form factors [Liu '20]



PRL 115, 222001 (2015) PHYSICAL REVIEW LETTERS week ending 27 NOVEMBER 2015

**Can Nonrelativistic QCD Explain the $\gamma\gamma^* \to \eta_c$ Transition Form Factor Data?**

PRL 119, 252001 (2017) PHYSICAL REVIEW LETTERS week ending 22 DECEMBER 2017

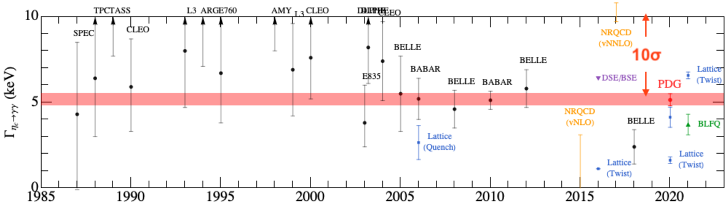
**Next-to-Next-to-Leading-Order QCD Corrections to the Hadronic Width of Pseudoscalar Quarkonium**

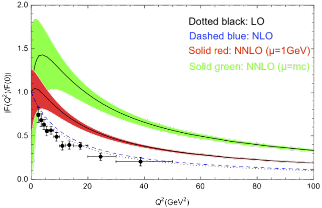
Feng Feng,[1,2] Yu Jia,[1,3,4] and Wen-Long Sang[5,*]

In our opinion, this may signal a profound crisis for the influential NRQCD factorization approach—whether it can be adequately applicable to charmonium decay or not. Our

## Why charmonium is so challenging?

- $\alpha_s \sim (0.3 - 0.6)$ is not that small -- non-perturbative effects
- $v_c^2 \sim 0.3$ is not that small -- relativistic effects
- $am_c \sim 0.5$ is not that small -- high order $O(a^2)$ effects



# A crisis in NRQCD for charmonium?

## Personal biased perspectives 贾宇老师，2019

Maybe Nature is just not so mercy to us:

The charm quark is simply not heavy enough to warrant the reliable application of NRQCD to charmonium, just like one cannot fully trus HQET to cope with charmed hadron

Symptom: mc is not much greater than $\Lambda_{QCD}$

Bigger value of $\alpha_s$ at charm mass scale

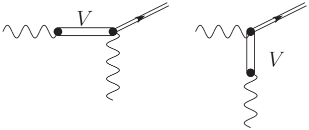
But we should still trust NRQCD to be capable of rendering qualitatively correct phenomenology for charmonium

We may need be less ambitious for soliciting precision predictions 54

# T$\gamma$P: vector meson dominance & light-cone dominance [Berger '87]

$Q^2 \to 0$

- Low $Q^2$: vector meson dominance [Sakurai '63, Novikov '78]



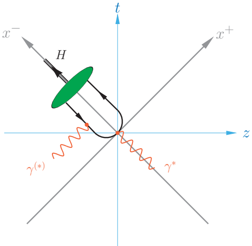

$$i\mathcal{M} \sim (Q^2 + M_V^2)^{-1} \underbrace{\psi(\vec{r} = 0)}_{\text{wave function at origin}}$$

$Q^2 \to \infty$

- Large $Q^2$: light-cone dominance [Lepage '80 & Chernyak '84]

$$i\mathcal{M}^{\mu\nu} = \int \mathrm{d}^4 x e^{iq\cdot z} \langle 0 | J^\mu(z) J^\nu(0) | P \rangle \sim \int \mathrm{d}x\, T_H(x, Q^2) \underbrace{\phi_P(x;\mu)}_{\text{light-cone distribution amplitude}}$$

*Large-$Q^2$ limit:* $z^2 \sim 1/Q^2 \to 0$ *(the light cone)* [Gribov '83, Nandi '07 & Li '09]



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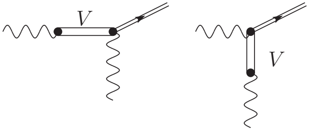
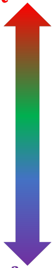



$\boldsymbol{Q^2 \to 0}$



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- Low $Q^2$: vector meson dominance [Sakurai '63, Novikov '78]

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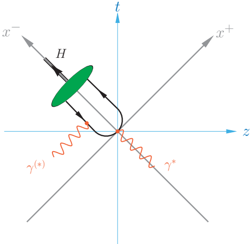

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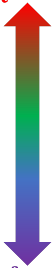

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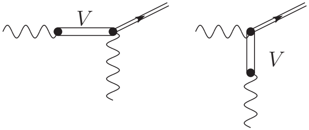
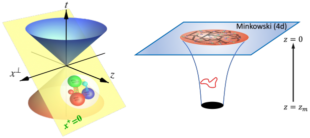



- Intermediate $Q^2$: get off the light cone ($z^2 = 0$) $\to$ light front ($z^+ = 0$)

$$i\mathcal{M}^{\mu\nu} \sim \int_0^1 \mathrm{d}x \int \frac{\mathrm{d}^2 k_\perp}{16\pi^3} T_H(x, \vec{k}_\perp; Q^2) \underbrace{\psi_P(x, \vec{k}_\perp; \mu)}_{\text{light-front wave function}}$$

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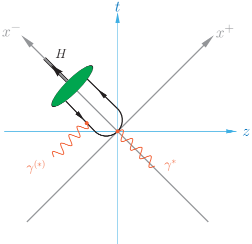
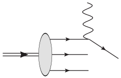
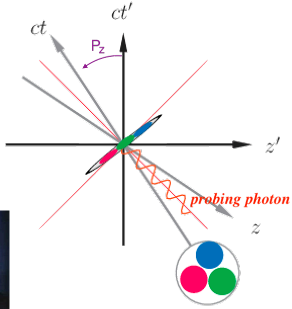


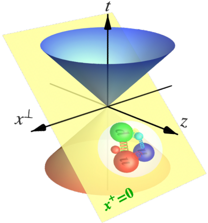
# Physics on the light front

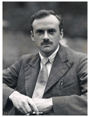
[Reviews: Brodsky '98, Burkardt '02, Bakker '14, Ji '21]

infinite momentum frame ($P_z \to \infty$) light front quantization ($x^+ = 0$)



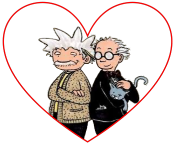


$$x^\pm = x^0 \pm x^3$$
$$P^\pm = P^0 \pm P^3$$
$$\underline{\mathcal{M}}^2 = P^+ \underline{P}^- - \vec{P}_\perp^2$$

$$i\frac{\partial}{\partial x^+}|\psi(x^+)\rangle = \frac{1}{2}\underline{P}^-|\psi(x^+)\rangle$$
$$\Downarrow$$
$$\underline{\mathcal{M}}^2|\psi_h(P,j,m_j)\rangle = M_h^2|\psi_h(P,j,m_j)\rangle$$

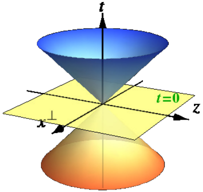


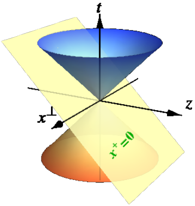
# Dirac's forms of relativistic dynamics

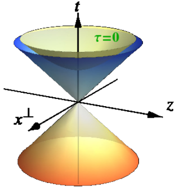
[Dirac, Rev. Mov. Phys. '49]

| | ***instant form*** | ***front form*** | ***point form*** |
|---|---|---|---|
| time variable | $t = x^0$ | $x^+ \triangleq x^0 + x^3$ | $\tau \triangleq \sqrt{t^2 - \vec{x}^2 - a^2}$ |

quantization surface



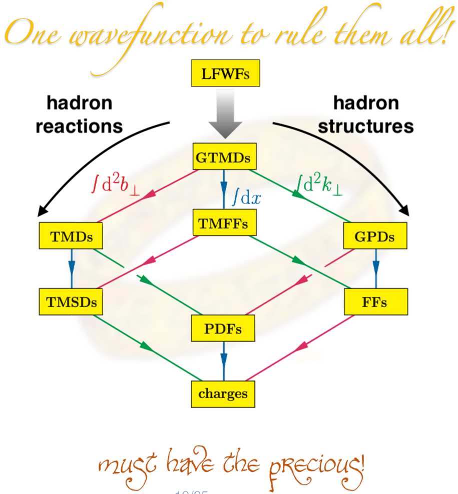




| | instant form | front form | point form |
|---|---|---|---|
| Hamiltonian | $H = P^0$ | $P^- \triangleq P^0 - P^3$ | $P^\mu$ |
| kinematical | $\vec{P}, \vec{J}$ | $\vec{P}^\perp, P^+, \vec{E}^\perp, E^+, J_z$ | $\vec{J}, \vec{K}$ |
| dynamical | $\vec{K}, P^0$ | $\vec{F}^\perp, P^-$ | $\vec{P}, P^0$ |
| dispersion relation | $p^0 = \sqrt{\vec{p}^2 + m^2}$ | $p^- = (\vec{p}_\perp^2 + m^2)/p^+$ | $p^\mu = m v^\mu \; (v^2 = 1)$ |

$P^\pm \triangleq P^0 \pm P^3$, $\vec{P}^\perp \triangleq (P^1, P^2)$, $x^\pm \triangleq x^0 \pm x^3$, $\vec{x}^\perp \triangleq (x^1, x^2)$, $E^i = M^{+i}$, $E^+ = M^{+-}$, $F^i = M^{-i}$, $K^i = M^{0i}$, $J^i = \frac{1}{2}\epsilon^{ijk} M^{jk}$.

# Light-front wave functions (LFWFs) [Reviews: Brodsky '98, Diehl '03, Lorcé '11]

LFWFs are frame independent and can directly access the partonic information of hadrons

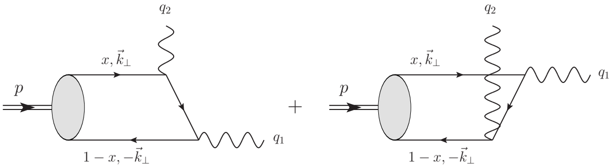



*Adapted from C. Lorcé*

*Hadron Physics without LFWFs is like Biology without DNA!*

*--- Stanley J. Brodsky*

# LFWF representation of two-photon transitions




- The amplitude can be accessed in light-cone perturbation theory and also through hadronic matrix elements, [Lepage '80, Feldmann '97, Kroll '10, Babiarz '19]

$$\varepsilon^*_\mu(q_1,\lambda_1)\varepsilon^*_\nu(q_2,\lambda_2)e_\alpha(p,\lambda)\mathcal{M}^{\mu\nu\alpha} = \varepsilon^*_\nu(q_2,\lambda_2)\langle\gamma^*(q_1,\lambda_1)|J^\nu(0)|H(p,\lambda)\rangle.$$

It is convenient to adopt a frame in which $q_1^- = q_2^+ = 0$, i.e. with manifest light-cone dominance.

- Example: LFWF representation of a pseudoscalar meson $(0^{-+})$,

$$F_{P\gamma}(Q^2) = e_f^2 2\sqrt{2N_C}\int\frac{\mathrm{d}x}{2\sqrt{x(1-x)}}\int\frac{\mathrm{d}^2k_\perp}{(2\pi)^3}\frac{\psi^*_{\uparrow\downarrow-\downarrow\uparrow/P}(x,\vec{k}_\perp)}{k_\perp^2+m_f^2+x(1-x)Q^2}+\cdots$$

Intuitively, this is the overlap of the photon wave function with the meson wave function. [Beuf '16, Lappi '20]

# Light-front dynamics $\neq$ NR dynamics w. relativistic corrections

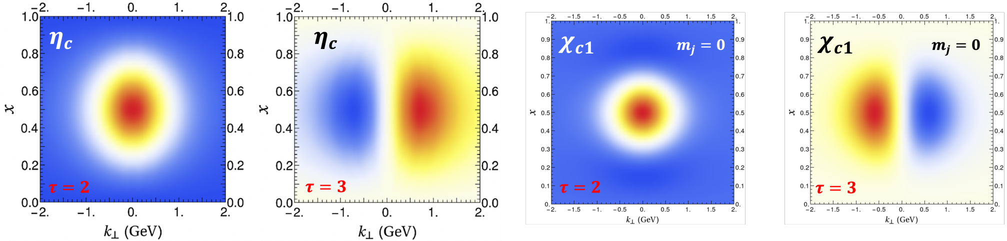
N.B. wave functions from non-relativistic dynamics with relativistic corrections in general are different from wave functions from relativistic dynamics, e.g. LFD.

- Parities in NRQM: $\mathsf{P} = (-1)^{L+1}, \mathsf{C} = (-1)^{L+S}$ are approximations since $L$ is not a good quantum number

  *NRQM + relativistic correction: spin-orbital coupling $\to$ partial wave mixing subject to parities*

- Parities in LFD: $m_{\mathsf{P}} = (-1)^{L_z+S+1}, \mathsf{C} = (-1)^{L_z+S+\ell}$ are exact

  *There exist leading-twist wavefunctions that are absent in NRQM (including relativistic corrections) due to parities*

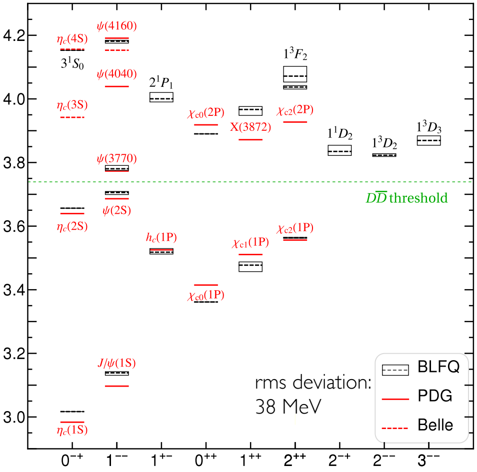



(a) $\psi_{\uparrow\downarrow-\downarrow\uparrow}(\vec{k}_\perp, x)$ (b) $\psi_{\downarrow\downarrow}(\vec{k}_\perp, x) = \psi^*_{\uparrow\uparrow}(\vec{k}_\perp, x)$ (c) $\psi_{\uparrow\downarrow-\downarrow\uparrow}(\vec{k}_\perp, x)$ (d) $\psi_{\downarrow\downarrow}(\vec{k}_\perp, x) = \psi^*_{\uparrow\uparrow}(\vec{k}_\perp, x)$

# Basis light-front quantization

[Vary et al. PRC '09; YL, Maris, Zhao, Vary, PLB '16]

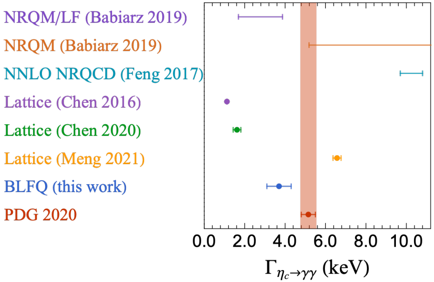
- Holographic light-front QCD confinement plus a longitudinal confinement [Review: Brodsky '14]
- Solved in basis function approach, $\Lambda_{\mathrm{UV}} \approx b\sqrt{N_{\mathrm{max}}}$.
- Two free parameters $m_c, \kappa$ fitted to the mass spectrum. Posterior rms deviation: $\lesssim$ 40 MeV
- Application to a variety of systems:
  - $c\bar{c}, b\bar{b}$: YL, PLB '16 & PRD '17
  - $b\bar{c}, b\bar{q}, c\bar{q}$: Tang, PRD '18 & EPJC '20
  - $q\bar{q}$: Jia, PRC '19; Qian, PRC '20; YL, '21
  - Baryons: Mondal, PRD '20; Xu, '21; Shuryak, PRD '21
- Access to a variety of observables
  - Form factors: YL, PRD '18; Mondal, PRD '20
  - (Semi-)leptonic decay: Li, PRD '18 & '19; Tang, PRD '21
  - PDFs/GPDs: Lan, PRL '19 & PRD '20; Adhikari, PRC '18 & '21
  - Diffractive production: Chen, PLB '17 & PRC '18

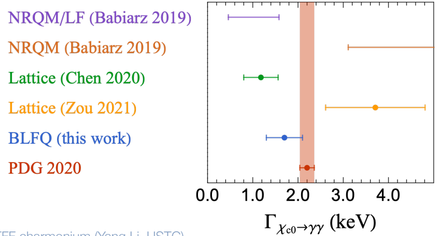


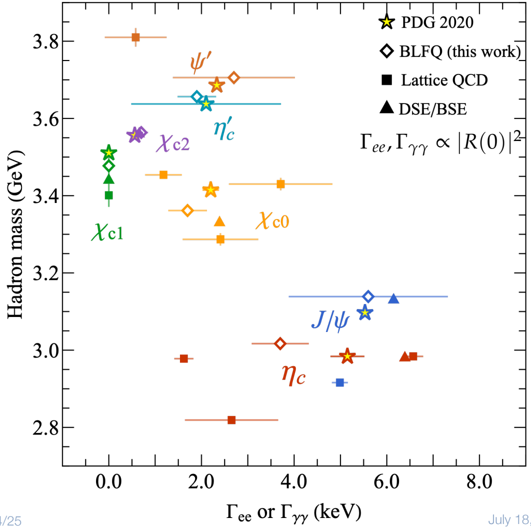
# Numerical results: diphoton widths

Our results are extremely competitive!

[Lattice: Dudek '06, Chen '16, Chen '20, Meng '21, Zou '21; DSE: Chen '16]



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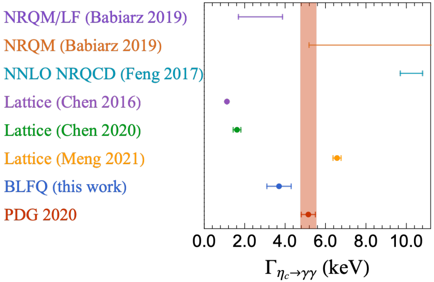
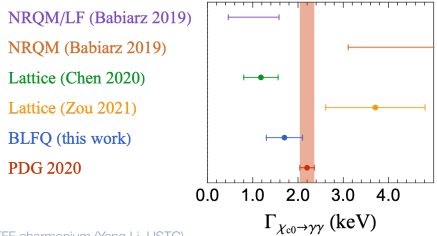
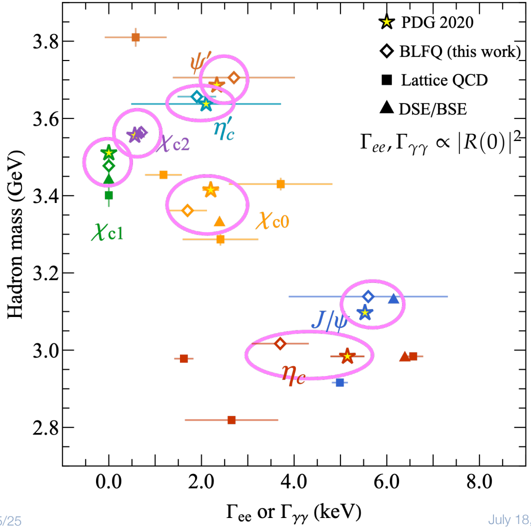


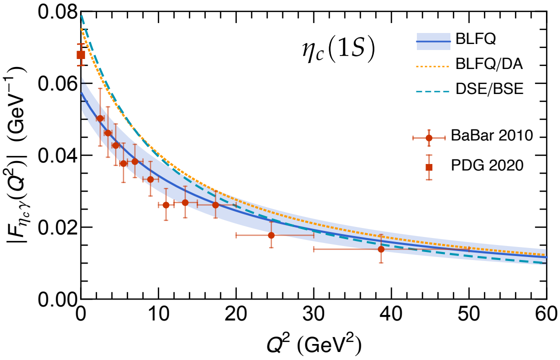
# Transition form factor: $\eta_c$

$$\mathcal{M}^{\mu\nu} = 4\pi\alpha_{\rm em}\varepsilon^{\mu\nu\rho\sigma}q_{1\rho}q_{2\sigma}F_{P\gamma\gamma}(q_1^2, q_2^2), \quad F_{P\gamma}(Q^2) \equiv F_{P\gamma\gamma}(Q^2, 0) = F_{P\gamma\gamma}(0, Q^2)$$

Diphoton width: $\Gamma_{\gamma\gamma} = \frac{\pi}{4}\alpha_{\rm em}^2 M_P^3 \big|F_{P\gamma\gamma}(0,0)\big|^2.$

[Lepage '81, Babiarz '19, Hoferichter '20]

$$F_{P\gamma}(Q^2) = e_f^2 2\sqrt{2N_C}\int\frac{\mathrm{d}x}{2\sqrt{x(1-x)}}\int\frac{\mathrm{d}^2k_\perp}{(2\pi)^3}\frac{\psi^*_{\uparrow\downarrow-\downarrow\uparrow/P}(x,\vec{k}_\perp)}{k_\perp^2 + m_f^2 + x(1-x)Q^2}$$




- BABAR data: $F_{\eta_c\gamma} \propto 1/(Q^2 + \Lambda^2)$, where the pole mass $\Lambda^2 = 8.5 \pm 0.6 \pm 0.7\,\mathrm{GeV}^2$; width $\Gamma_{\gamma\gamma} = 5.12(53)\,\mathrm{keV}$.
- BLFQ: using $N_{\max} = 8$, corresponding to $\mu \approx 2m_c$. Basis sensitivity band is taken as the difference between the $N_{\max} = 8, 16$ results.
- BLFQ/DA: prediction using the LCDA obtained first from the LFWF
- Theoretical prediction in good agreement with both the width and the form factor.

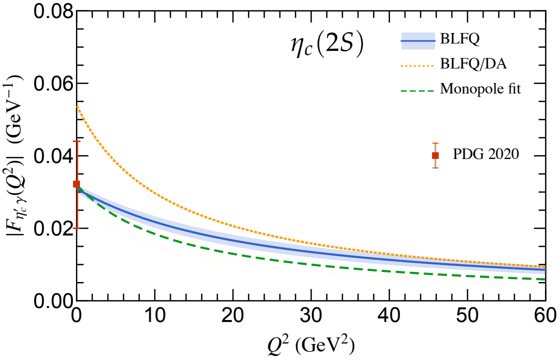
[DSE/BSE: Chen, PRD '17]

# Transition form factor: $\eta_c$

$$\mathcal{M}^{\mu\nu} = 4\pi\alpha_{\mathrm{em}}\varepsilon^{\mu\nu\rho\sigma}q_{1\rho}q_{2\sigma}F_{P\gamma\gamma}(q_1^2, q_2^2), \quad F_{P\gamma}(Q^2) \equiv F_{P\gamma\gamma}(Q^2, 0) = F_{P\gamma\gamma}(0, Q^2)$$

Diphoton width: $\Gamma_{\gamma\gamma} = \frac{\pi}{4}\alpha_{\mathrm{em}}^2 M_P^3 \left|F_{P\gamma\gamma}(0,0)\right|^2$.

$$F_{P\gamma}(Q^2) = e_f^2 2\sqrt{2N_C}\int \frac{\mathrm{d}x}{2\sqrt{x(1-x)}}\int \frac{\mathrm{d}^2k_\perp}{(2\pi)^3}\frac{\psi^*_{\uparrow\downarrow-\downarrow\uparrow/P}(x,\vec{k}_\perp)}{k_\perp^2 + m_f^2 + x(1-x)Q^2}$$




- No experimental measurement yet.
- A monopole fit using $\Lambda^2 = M_{\psi'}^2$ is included for comparison.
- Note that a VMD prediction requires the off-shell coupling $g_V(Q^2) = V_{PV\gamma}(Q^2)$: [Lakhina '06]

$$F_{P\gamma}^{(\mathrm{VMD})}(Q^2) = \sum_V \frac{e_f^2 f_V}{1 + \frac{M_P}{M_V}}\left[\frac{g_V(0)}{M_V^2 + Q^2} + \frac{g_V(Q^2)}{M_V^2}\right]$$

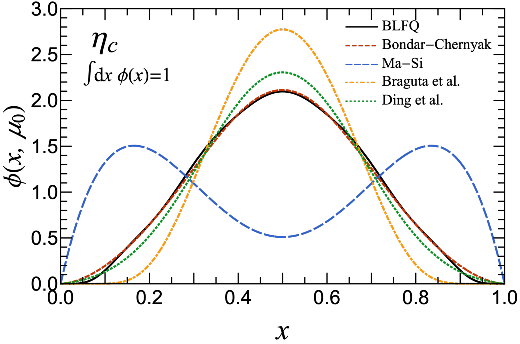
# Light cone distribution amplitude (LCDA)

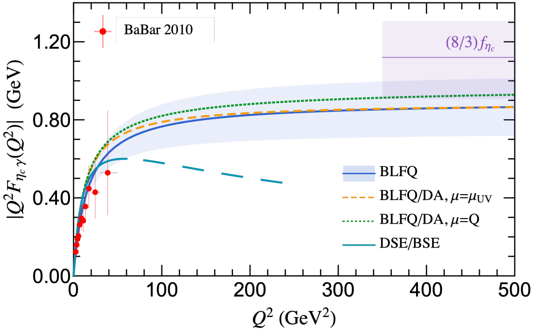
At large-$Q^2$, viz. $Q^2 + \langle m_f^2/x(1-x)\rangle \gg \langle k_\perp^2/x(1-x)\rangle$,

$$F_{P\gamma}(Q^2) \approx e_f^2 f_P \int_0^1 \mathrm{d}x \frac{\phi_P(x,\mu)}{x(1-x)Q^2 + m_f^2} \xrightarrow{Q\to\infty} \frac{6e_f^2 f_P}{Q^2}.$$

- LCDA plays a pivotal role in hard exclusive charmonium production. [See, e.g., Braguta '12]
- Our LCDA agrees with the Bondar-Chernyak model. Both fit the BABAR normalized TFF well.

[LCDAs: Ma '04, Bondar '05, Braguta '07, Ding '16]



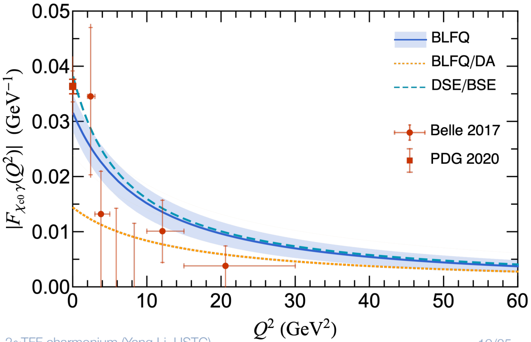



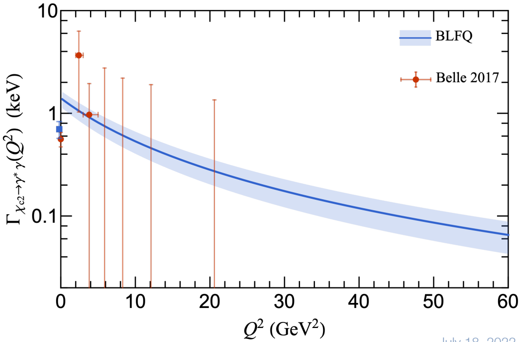
# Transition form factor: $\chi_{cJ}$

[Babiarz '20, Hoferichter '20; DSE/BSE: Chen '17]

$$\mathcal{M}^{\mu\nu}_{S\to\gamma\gamma} = 4\pi\alpha_{\rm em}\Big\{\big[(q_1\cdot q_2)g^{\mu\nu} - q_2^\mu q_1^\nu\big]F_1^S(q_1^2,q_2^2) + \frac{1}{M_S^2}\big[q_1^2q_2^2g^{\mu\nu} + (q_1\cdot q_2)q_1^\mu q_2^\nu - q_1^2q_2^\mu q_2^\nu - q_2^2q_1^\mu q_1^\nu\big]F_2^S(q_1^2,q_2^2)\Big\}$$

Single-tag TFF: $F_{S\gamma}(q^2) = F_1^S(q^2,0) = F_1^S(0,q^2)$. Width $\Gamma_{\gamma\gamma} = \frac{\pi\alpha_{\rm em}^2}{4}M_S^3|F_{S\gamma}(0)|^2$. Belle provides the first measurement of the TFF, albeit with limited statistics. [Belle, PRD 2017]

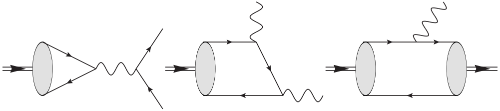


# Radiative transitions

Leptonic and radiative transitions probe the fundamental structure of the hadrons:

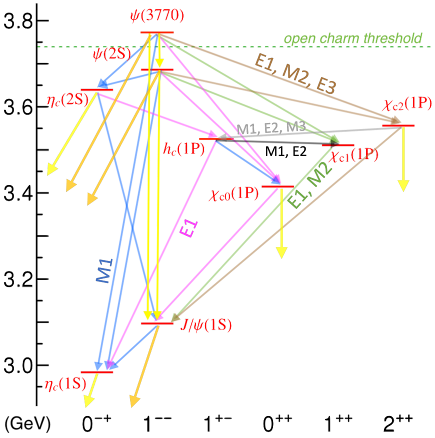
[Review: Barnes & Yuan, Int. J. Mod. Phys. A 2009]



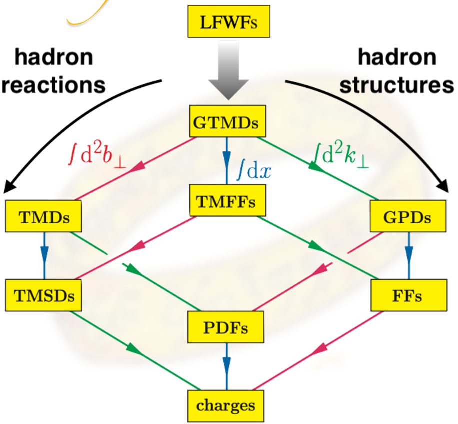
| decay width (keV) | | $\Gamma_{ee}$ | $\Gamma_{\gamma\gamma}$ | $\Gamma_{\eta_c\gamma}$ | $\Gamma_{J/\psi\gamma}$ |
|---|---|---|---|---|---|
| $\eta_c$ | PDG | - | 5.15(35) | | |
| | BLFQ | - | 3.7(6) | | |
| $J/\psi$ | PDG | 5.53(10) | - | 1.6(4) | |
| | BLFQ | 5.7(1.9) | - | 2.6(1) | |
| $\chi_{c0}$ | PDG | - | 2.1(1.6) | - | $15(1)\times 10^3$ |
| | BLFQ | - | 1.9(4) | - | in progress |
| $\chi_{c1}$ | PDG | - | - | - | 288(16) |
| | BLFQ | - | - | - | in progress |
| $\vdots$ | | | | | |

[YL, PRD '17; Li, PRD '18; Chen, in progress]

[PDG, PTEP '20 + '21(update)]

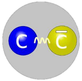


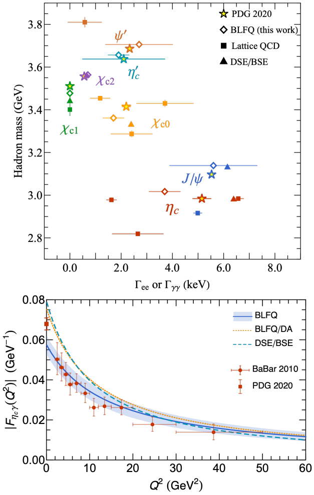
# *One wavefunction to rule them all!*



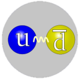
# The state of the art examples from basis light-front quantization

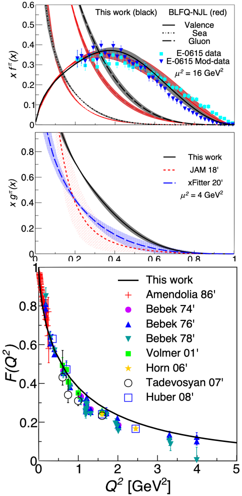
[Li, PRD '22]



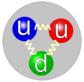



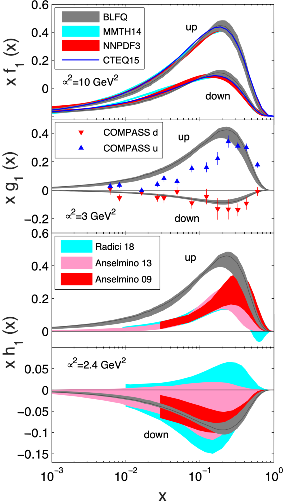

[Lan, PRL '19]



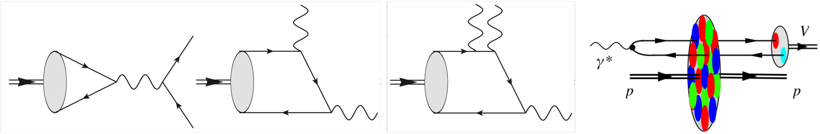

[Mondal, PRD '20]



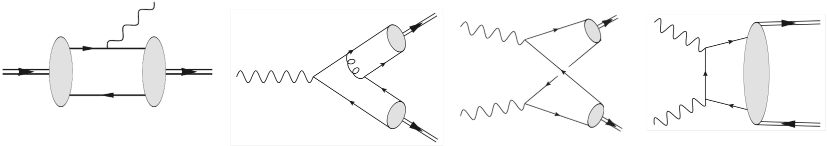



# Parton structures from $e^+e^-/\gamma\gamma$ collisions

- Leptonic & radiative decay, diffractive meson production $\langle\gamma^*|J(x)|H\rangle$




- Space-like form factors, radiative transitions $\langle H'|J(x)|H\rangle$
- Time-like form factors, double hadron photoproductions
- Two-photon productions, generalized distribution amplitude, transition distribution amplitude

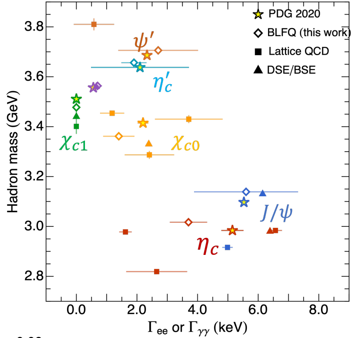


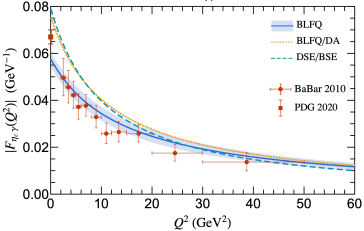
# Summary

- Light-front Hamiltonian formalism provides unique tools to access the hadronic observables
  - Light-cone dominance
  - Collinear factorization and $k_T$ factorization
- We computed the two-photon width and transition form factors of charmonia within the basis light-front quantization approach.
  - Excellent agreements with the available experimental measurements.
  - No parameters are dialed to obtain these results.
  - Reveal relativistic nature of charmonium system
- The obtained wave functions await further experimental measurements and further applications.

Based on: YL, M. Li (李枚键) and J.P. Vary, Phys. Rev. D letter **105** (2022); arXiv:2111.14178 [hep-ph]

LFWFs available on Mendeley Data



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Thank you for your attention.